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# A review on the mechanics of nanostructures

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## Abstract

Understanding the mechanical behaviour of nanostructures is of great importance due to their applications in nanodevices such as in nanomechanical resonators, nanoscale mass sensors, electromechanical nanoactuators and nanogenerators. Due to the difficulties of performing accurate experimental measurements at nanoscales and the high computational costs associated with the molecular dynamics simulations, the continuum modelling of nanostructures has attracted a considerable amount of attention. Since size influences have a crucial role in the mechanics of structures at nanoscale levels, classical continuum-based theories have been modified to incorporate these effects. Among various modified continuum-based theories, the nonlocal elasticity and the nonlocal strain gradient elasticity have been employed to estimate the mechanical behaviour of nanostructures. In this review paper, first these two modified elasticity theories are briefly explained. Then, the nonlocal motion equations for different nanostructures including nanorods, nanorings, nanobeams, nanoplates and nanoshells are derived. Several papers which reported on the size-dependent mechanical behaviour of nanostructures using modified continuum models are reviewed. Furthermore, important results reported on the vibration, bending and buckling of nanostructures as well as the results of size-dependent wave propagation analyses are discussed.

*Keywords: Nanostructure, Size-dependent; Modified continuum models; Mechanical behaviour*

## 1. Introduction

Nanoscale structures including nanoscale rods [1], rings [2], beams [3-8], plates [9, 10], and shells [11, 12] have been utilised as the fundamental structural parts of many nanoelectromechanical systems (NEMS). Nanomechanical resonators [13, 14], nanoscale mass sensors [15], electromechanical nanoactuators [16], and nanoenergy harvesters [17] are salient examples of these NEMS-based devices. These valuable nanoscale devices have broad applications in different areas of nanotechnology such as nanoelectronics, nanomachines and nanomedicine. To achieve a better performance for the nanodevice, a better understanding of the mechanical characteristics of nanostructures as these ultrasmall structures are usually subject to mechanical loads, pressure or stresses. In addition, in nanodevices such as nanoscale generators [18, 19], the mechanical energy is converted into electricity, hence an analysis on the mechanical behaviour is essential.

Since performing an accurate experimental measurement at nanoscale levels is challenging, continuum-based modelling and molecular dynamics (MD) simulations of nanostructures have attracted a considerable amount of attention. Using continuum-based models and the results of MD simulations, the number of required experimental measurements can be reduced. Compared to MD simulations, the continuum modelling of nanostructures is less computationally expensive. Particularly, performing MD simulations on a nanostructure with a large number of molecules requires a high computational effort. Using continuum models, the mechanical characteristics can be formulated and estimated. In this way, the computational costs of MD simulations can be reduced by eliminating unnecessary simulations. In addition, the continuum-based modelling of

nanostructures can help us to better understand the results of experimental measurements or molecular dynamics.

Scale effects have a crucial role to play in the mechanics of nanostructures, as opposed in macrostructures [20-26]. Thus, traditional continuum-based theories, which are scale-free, have been modified in order to capture size effects. Various size-dependent theories for examining the mechanical characteristics of nanostructures have been introduced in recent years. Since the mechanical behaviour of structures at microscale levels [27-39] is different from that observed at nanoscale levels, the modified continuum-based theories of microstructures are different from those of nanostructures [40]. In general, the structural stiffness hardening is observed at microscale levels whereas the mechanics of nanostructures is usually governed by the stiffness softening. Therefore, size-dependent models including the couple stress [41-51] and strain gradient elasticities [52-54] are often used to analyse the mechanical behaviour of microstructures including microbeams, microbars and microplates while the nonlocal elasticity theory [55-58] is applied to nanoscale structures. However, to have a more general size-dependent continuum-based model capable of predicting size effects at different small scales, a combination of these modified elasticity theories [59] can be employed.

This review article is organised as follows: In Section 2, concise information is given about different size-dependent elasticity theories utilised for investigating the mechanical characteristics of structures at nanoscale levels including the pure nonlocal and nonlocal strain gradient elasticities. In Section 3, first the size-dependent motion equations of various types of nanoscale structures such as nanoscale rods, rings, beams, plates and shells are developed via the nonlocal elasticity. Then, studies on the size-dependent modelling of the mechanical

behaviour of these structures are reviewed; particular attention is paid to the size-dependent bending, buckling and vibration of nanoscale structures as well as size-dependent wave propagations in these small-scale structures. Finally, Section 4 concludes on the size-dependent continuum theories of nanostructures, and the most important findings to date are highlighted.

## **2. Size-dependent continuum mechanics**

In this section, size-dependent [60-69] elasticity theories including the nonlocal elasticity and the nonlocal strain gradient elasticity, which are commonly applied to nanoscale structures, are reviewed. Firstly, the basic concept of the nonlocal elasticity is clarified, and then both the integral and differential nonlocal constitutive relations are discussed. Finally, the theory of the nonlocal strain gradient elasticity is introduced.

### *2.1. Nonlocal elasticity theory*

The nonlocal elasticity was introduced by Eringen [70, 71] almost two decades before the invention of carbon nanotubes (CNTs). However, this valuable theory did not attract much attention until the synthesis of nanostructures such as CNTs and graphene sheets (GSs) emerged. Peddieson et al. [72] first suggested that the theory can be used to analyse the size-dependent mechanical response of nanostructures. In the classical elasticity theory, which is not able to predict size effects, the stress at a spot is only dependent on the strain at that spot. By contrast, in the nonlocal elasticity, strains at all spots affect the stress at one arbitrary spot as shown in Fig. 1. This basic assumption allows this theory to capture intermolecular interactions, leading to

a size-dependent theory of elasticity. Ignoring body forces, the nonlocal integral constitutive relation is given by

$$\sigma_{ij}^{nl} = \iiint_V \varphi(|\mathbf{x} - \mathbf{x}'|, \eta) \sigma_{ij}^l dV, \quad (1)$$

where  $\sigma_{ij}^{nl}$ ,  $\sigma_{ij}^l$ ,  $\varphi$  and  $\eta$  stand for the nonlocal stress, local stress, kernel function and small-scale coefficient, respectively;  $|\mathbf{x} - \mathbf{x}'|$  is the distance from  $\mathbf{x}$  to  $\mathbf{x}'$ , and  $V$  denotes the volume of the body. The nonlocal coefficient is expressed as

$$\eta = \frac{e_0 a}{L}, \quad (2)$$

in which  $e_0$ ,  $a$  and  $L$  are respectively the calibration coefficient, and internal and external characteristic lengths. Each nanostructure has internal and external characteristic lengths. For example, for carbon nanotubes, the c-c bond length is commonly chosen as the internal characteristic length (see Fig. 2). The calibration coefficient is obtained either from experimental measurements or molecular dynamics (MD). The classical (local) stress is obtained as

$$\sigma_{ij}^l = C_{ijkl} \varepsilon_{kl}, \quad (3)$$

where  $C_{ijkl}$  and  $\varepsilon_{kl}$  stand for the elasticity tensor and the strain tensor, respectively.

Since the nonlocal constitutive Eq. (1) must reduce to that of the classical elasticity theory for very large external characteristic length, the kernel function (nonlocal modulus) has the following property

$$\lim_{\eta \rightarrow 0} \varphi(|\mathbf{x} - \mathbf{x}'|, \eta) = \delta(|\mathbf{x} - \mathbf{x}'|). \quad (4)$$

Here  $\delta$  denotes the Dirac delta. Eringen [73] introduced some kernel functions for nonlocal problems. One of the most popular kernel functions is given by

$$\varphi(|\mathbf{x}|, \eta) = (2\pi L^2 \eta^2)^{-1} K_0 \left( \frac{\sqrt{\mathbf{x} \cdot \mathbf{x}}}{L\eta} \right), \quad (5)$$

in which  $K_0$  denotes the modified Bessel function. Since the implementation of the integral nonlocal constitutive equation (i.e. Eq. (1)) in formulating the mechanics of nanostructures is difficult, a nonlocal operator ( $L_{nl}$ ) with the following property is introduced

$$L_{nl}\varphi(|x-x'|, \eta) = \delta(|x-x'|). \quad (6)$$

Applying the nonlocal operator to Eq. (1), one can obtain

$$L_{nl}\sigma_{ij}^{nl} = \sigma_{ij}^l. \quad (7)$$

Using the above equations, Eringen [71, 73] obtained the following relation for the nonlocal operator

$$L_{nl}(\ast) = [1 - (e_0 a)^2 \nabla^2](\ast). \quad (8)$$

Here  $\nabla^2$  stands for the Laplace operator. Equations (7) and (8) are extensively used to develop size-dependent continuum models in order to estimate the mechanical response of nanostructures.

## 2.2. *Nonlocal strain gradient elasticity*

There are two limitations associated with the nonlocal elasticity theory. Firstly, nonlocal effects disappear after a certain length. For instance, scale effects predicted by the nonlocal elasticity on the axial vibration of uniform nanorods disappear for  $L > 20$  nm [74]. Secondly, the nonlocal elasticity can only predict the stiffness softening of small-scale structures. However, stiffness hardening has been observed in some small-scale structures, especially at higher lengths. This stiffness hardening can be estimated incorporating surface effects [75-80] or strain gradients [52, 53, 81-86]. For example, it was found that the pure nonlocal plate model cannot completely predict the buckling instability of circular graphene sheets subject to axisymmetric radial loading [87] by employing MD simulations. To overcome the shortcomings of the nonlocal elasticity, Lim et al. [59] introduced a nonlocal strain gradient theory (NSGT) using two kernel functions. The new theory is able to describe both stiffness softening and hardening at small-scale levels. In addition, the scale effect predicted by the NSGT appears in a wider range of lengths in comparison with nonlocal effects. However, the computational costs of the nonlocal elasticity is less than those of the NSGT due to the fact that strain gradient terms are also incorporated.

## 3. **Types of different nanostructures**

In the following sub-sections, the size-dependent continuum models of various types of nanoscale structures including nanorods, nanorings, nanobeams, nanoplates and nanoshells as well as the literature on the mechanics of these nanostructures are reviewed. Furthermore, size-dependent differential equations for the mechanical behaviours of these structures such as their



buckling, vibration, bending and wave propagation responses are presented via the nonlocal elasticity. Various types of nanostructures, involving nanorods, nanorings, nanobeams, nanoplates, and nanoshells, are considered in this paper (see Fig. 3).

### 3.1. Nanorods

In this section, modified continuum models reported on the mechanical behaviour of nanorods are reviewed. Nanorods [88] are one-dimensional nanoscale structures which can be made by various techniques such as vapour-phase transport [1], hydrothermal synthesis [89] and seed-mediated growth [90] (see Fig. 4). The length of nanorods can vary from 1 nm to 3000 nm [91]. These small-scale structures have been extensively utilised in various devices including nanosensors [92], drug delivery systems [93] and solar cells [94]. To better design nanosystems using nanorods, it is advised to enhance knowledge about the mechanics of these structures since the overall performance of a nanosystem is affected by the mechanical characteristics of its parts.

#### 3.1.A. Nonlocal rod model

Aydogdu [74] developed a nonlocal model for the linear longitudinal vibration of nanoscale rods. Following him, one can write the nonlocal stress of nanorods as

$$\sigma_{xx}^{nl} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}^{nl}}{\partial x^2} = E \varepsilon_{xx}, \quad (9)$$

where  $E$  is the elasticity modulus. Using Eq. (9), the force resultant of nanorods (i.e.

$N_{xx} = \int_A \sigma_{xx}^{nl} dA$ ) can be expressed as

$$N_{xx} - (e_0 a)^2 \frac{\partial^2 N_{xx}}{\partial x^2} = EA \frac{\partial u}{\partial x}, \quad (10)$$

where  $A$  and  $u$  denote the area of the rod cross-section and the axial displacement, respectively.

Now employing the Hamilton principle, one can obtain the motion equation of nanorods as

$$\frac{\partial N_{xx}}{\partial x} = m \frac{\partial^2 u}{\partial t^2}. \quad (11)$$

Here  $m$  represents the mass per unit length of the nanorod. Using Eqs. (10) and (11), the following explicit relation is obtained for the stress resultant

$$N_{xx} = EA \frac{\partial u}{\partial x} + (e_0 a)^2 m \frac{\partial^3 u}{\partial x \partial t^2}. \quad (12)$$

Substituting Eq. (12) into Eq. (11), one obtains

$$EA \frac{\partial^2 u}{\partial x^2} + m (e_0 a)^2 \frac{\partial^4 u}{\partial x^2 \partial t^2} = m \frac{\partial^2 u}{\partial t^2}. \quad (13)$$

Equation (13) governs the axial vibration of nanoscale rods incorporating size effects. This equation was first derived by Aydogdu [74]. He solved the equation analytically and presented explicit expressions for two different boundary conditions. Figure 5 shows the ratio of the local natural frequency to the nonlocal one for various scale parameters. The frequency ratios are calculated for clamped-clamped (C-C) and clamped-free (C-F) nanorods. It is found that for small lengths, scale effects are noticeable while the nonlocal and local frequencies are approximately the same after a certain length ( $L > 20$  nm). More recently, the nonlocal rod model has been employed to analyse the longitudinal free vibration of nanorods for different boundary conditions including an attached mass and an attached spring [95]; it was found that the mass attachment reduces the axial frequency of nanorods.

### 3.1.B. *Size-dependent mechanics of nanorods*

There are various types of nanoscale rods such as uniform, non-uniform and nonhomogeneous. In addition to simple uniform nanorods, the axial vibrations of tapered nanorods [96] and double-nanorod systems [97] were also investigated in the literature. Moreover, the axial vibration of nonhomogeneous rods at nanoscale levels was studied utilising the nonlocal elasticity [98, 99]; shown was that the material non-homogeneity can greatly affect the axial vibration of nanorods. Depending on the value of elasticity modulus ratio, the natural frequency of nanorods can decrease or increase with increasing power-law exponent [99].

In addition to the axial vibration of nanorods, other mechanical responses of these small-scale structures have been also investigated using size-dependent continuum models. For instance, wave propagations in nanorods were studied via help of the nonlocal elasticity [100, 101]; it was reported that the scale parameter greatly affects the wave propagation in nanorods. The size coefficient causes a certain region associated with the band gap in longitudinal wave modes. The nonlocal elasticity was also employed for analysing the size-dependent torsion of cracked nanorods [102]; the presence of a circumferential crack reduces the natural frequencies.

The majority of size-dependent continuum models of nanorods have been developed via the nonlocal theory of elasticity. However, more recently, the NSGT has been employed for describing the longitudinal vibration [103] and tension [104] of nanorods; the modified rod model was successfully calibrated employing MD results.

### 3.2. Nanorings

Another nanoscale structure with a remarkable potential applications in nanoelectromechanical systems (NEMS) is nanorings. Figure 6 illustrate a system of circular nanorings as well as a single nanoring. Compared to nanobeams and nanoplates, few theoretical studies have been reported on the mechanical behaviour of nanorings using size-dependent continuum models. Wang and Duan [105] developed a nonlocal model to explore the oscillations of nanoscale rings; exact results were obtained for the size-dependent natural frequencies. Assuming the flexural vibration of nanorings without extension, the nonlocal differential equation is obtained as [105]

$$\frac{\partial^6 v}{\partial \theta^6} + 2 \frac{\partial^4 v}{\partial \theta^4} + \frac{\partial^2 v}{\partial \theta^2} = \frac{mR^4}{EI} \left[ \frac{e_0 a}{R} \frac{\partial^6 v}{\partial t^2 \partial \theta^4} - \left( 1 + \frac{e_0 a}{R} \right) \frac{\partial^4 v}{\partial t^2 \partial \theta^2} + \frac{\partial^2 v}{\partial t^2} \right], \quad (14)$$

where  $v$ ,  $R$  and  $\theta$  are the tangential displacement, the radius of the nanoring and the angle between the horizontal line and the line drawn from the ring centre, respectively;  $E$ ,  $m$ ,  $I$  and  $e_0 a$  are the elasticity modulus, mass per unit length, inertia moment and the nonlocal parameter, respectively. In a paper by Wang and Duan [105], the oscillation of nanoscale arches (see Fig. 7) was also examined using the nonlocal elasticity. The variation of the frequency parameter versus the opening angle ( $2\beta$ ) for various nonlocal parameters ( $\alpha = e_0 a/R$ ) for (a) asymmetric and (b) symmetric modes is plotted in Fig. 8. The frequency parameter is defined as  $\Omega = mR^4 \omega^2 / EI$  where  $\omega$  is the dimensional natural frequency of the nanosystem. Increasing opening angle reduces the natural frequency of nanoarches. In addition, stronger nonlocal effects lead to lower natural frequencies since increasing nonlocal parameter results in a reduction in the stiffness of nanostructures.

In addition to the above-mentioned valuable study, the nonlocal elasticity theory was also utilised in Refs. [106-108] in order to derive size-dependent differential equations for investigating the mechanical behaviour of nanoscale rings. Moosavi et al. [106] developed a nonlocal shear deformation theory of rings for the in-plane free vibrations of nanorings; they found that the change in the natural frequency obtained by the classical theory and the shear deformation one is significant for small radii and larger nonlocal parameters. Furthermore, the size-dependent buckling of nanorings and nanoarches was analysed via help of the nonlocal elasticity in Refs. [107] and [108]; increasing scale parameter reduces the buckling force.

### 3.3. *Nanobeams*

Nanobeams [109-111] such as carbon nanotubes, silicon and silver nanobeams have various promising applications in different nanoscale devices such as small-scale mechanical sensors [15], resonators [13, 14] and actuators [16] (see Fig. 9). Since the small-scale system operates based on mechanical mechanisms in these applications, understanding the size-dependent mechanical characteristics of nanoscale beams is importance. In early studies on the mechanics of nanoscale beams, especially carbon nanotubes, size effects were not taken into consideration [112-115]. For the first time, Peddieson et al. [72] utilised the nonlocal continuum mechanics so as to capture size effects on the bending of nanobeams; particularly size-dependent bending of cantilever nanobeams was examined because of their wide applications in nanoscale actuators. In the following, the bending, vibration and buckling of nanobeams as well as the wave propagation in them are reviewed. Both size-dependent linear and nonlinear studies are considered.

### 3.3.A. Nonlocal beam model

Applying the nonlocal theory, the modified constitutive equation of a nanoscale beam can be expressed as

$$\left[1 - (e_0 a)^2 \nabla^2\right] \sigma_{xx} = E \varepsilon_{xx}. \quad (15)$$

On the other hand, applying the theory of Euler–Bernoulli beams, the axial strain is given by

$$\varepsilon_{xx}(x, z, t) = \frac{\partial u(x, t)}{\partial x} - z \frac{\partial^2 w(x, t)}{\partial x^2}, \quad (16)$$

where  $w$  and  $u$  indicate the mid-surface transverse and axial displacements of the nanobeam, respectively [116]. The force and couple stress resultants of nanobeams are defined as

$$N_{xx} = \int_A \sigma_{xx} dA, \quad M_{xx} = \int_A z \sigma_{xx} dA, \quad (17)$$

in which  $A$  is the cross-sectional area. For nanobeams, the following equations are derived via Hamilton's principle

$$\frac{\partial N_{xx}}{\partial x} = m \frac{\partial^2 u}{\partial t^2}, \quad (18)$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w}{\partial x} \right) + q = m \frac{\partial^2 w}{\partial t^2}. \quad (19)$$

Here  $m$  and  $q$  represent the mass per unit length and the transverse loading, respectively. In view of Eq. (15), one can write

$$\left[1 - (e_0 a)^2 \nabla^2\right] M_{xx} = -EI \frac{\partial^2 w}{\partial x^2}. \quad (20)$$

Using Eqs. (18)-(20), the following differential equation is derived for the linear transverse vibration of nonlocal beams subject to an external loading

$$\begin{aligned}
& -EI \frac{\partial^4 w}{\partial x^4} + \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w}{\partial x} \right) - (e_0 a)^2 \frac{\partial^3}{\partial x^3} \left( N_{xx} \frac{\partial w}{\partial x} \right) \\
& + q - (e_0 a)^2 \frac{\partial^2 q}{\partial x^2} = m \frac{\partial^2 w}{\partial t^2} - m (e_0 a)^2 \frac{\partial^4 w}{\partial x^2 \partial t^2}.
\end{aligned} \tag{21}$$

### 3.3.B. Size-dependent bending of nanobeams

Different modified beam models such as Reddy, Timoshenko, Euler–Bernoulli and Levinson were proposed for investigating the *bending* of nanoscale beams via the nonlocal continuum mechanics [56, 117-119]. Figure 10 shows the ratio of the maximum deflection of nanobeams under uniformly transverse loading obtained by the Timoshenko theory to that of the Euler–Bernoulli one with the ratio of the length ( $a$ ) to the internal characteristic length ( $l_i$ ) for various boundary conditions and various calibration coefficients ( $e_0$ ) [117]. In all cases, the influence of the calibration coefficient disappears after a certain length. Furthermore, explicit expressions were obtained in Refs. [120-122] in order to analyse the linear bending of nanobeams using the nonlocal elasticity as a size-dependent theory. The influence of the surface energy on the bending of nanobeams was also studied in the literature [79, 123]; it was found that the surface influence is more significant for slender nanobeams.

In addition, the nonlocal elasticity has been utilised to explore the size-dependent bending behaviour of *non-homogeneous* nanobeams [124, 125]. More recently, a NSGT-based beam model has been proposed by Li et al. [126] for the mechanical behaviour of non-homogeneous nanoscale beams. Figure 11 indicates the maximum deflection of a non-homogeneous nanobeam with simply supported boundary conditions subject to sinusoidal applied load in the transverse direction. Different stain gradient coefficients (i.e.  $\zeta = \ell/L$  in which  $\ell$  is the strain gradient

parameter) and various nonlocal coefficients (i.e.  $\tau = e_0 a/L$ ) are considered. Increasing strain gradient parameter reduces the maximum deflection of non-homogeneous nanobeams since higher strain gradient parameters increase the stiffness of nanostructures. By contrast, the maximum deflection notably increases with increasing nonlocal coefficient.

In addition to linear size-dependent models, *nonlinear* studies have been also reported on the static behaviour of nanoscale beams [127-129]. Reddy [127] presented both classical and shear deformable beam models incorporating the geometric nonlinearity as well as the size effects by nonlocal elasticity as well as von Kármán's assumptions. Moreover, a nonlinear size-dependent finite element formulation incorporating both surface and nonlocal effects was proposed by Preethi et al. [128] via use of the Timoshenko theory of beams. The nonlinear bending of nanoscale non-homogeneous beams has been lately examined by Li and Hu [129] using the NSGT as a size-dependent elasticity theory.

More recently, an *integral* size-dependent formulation has been developed by Fernández-Sáez et al. [130] so as to describe the bending of nanobeams using the nonlocal integral constitutive relation and the Euler–Bernoulli theory of beams. Using the integral nonlocal formulation, the paradox observed in cantilever nanobeams when the differential nonlocal elasticity is used (namely, the increasing effect of the nonlocal parameter on the nanobeam stiffness) was resolved as shown in Fig. 12. Moreover, Tuna and Kirca [131] examined the static deformation of nanobeams using the nonlocal integral model; exact results were obtained for both Euler–Bernoulli and Timoshenko nanobeams.



### 3.3.C. Size-dependent buckling of nanobeams

Size-dependent elasticity models have been also proposed for the *buckling* of nanoscale beams, especially carbon nanotubes. The majority of continuum models have been developed using the nonlocal elasticity [132-139]. For instance, Sudak [140] explored the linear stability of multi-walled carbon nanotubes (MWCNTs) via help of the nonlocal elasticity; nonlocal influences have a crucial role to play in the buckling of MWCNTs. In addition, Wang et al. [141] introduced a linear nonlocal theory for the stability of single-walled carbon nanotubes (SWCNTs). The variation of the nonlocal-to-local buckling ratio with the length is plotted in Fig. 13 for different nonlocal parameters. As the length increases, the effect of the length scale significantly decreases. Furthermore, higher values of  $e_0a$  reduces the buckling load ratio since the difference between the two theories increases when the nonlocal influence becomes stronger. Figure 14 illustrates the change of the buckling ratio with the length for different model numbers. It is observed that the influence of size is greater for higher mode numbers. This is due to the fact that at higher buckling modes, the interaction between molecules increases.

At nanoscale levels, the surface-to-bulk ratio of structures is high, and thus *surface influences* on the mechanical characteristics of nanostructures become important. The surface influence on the stability analysis of nanoscale beams has been investigated using modified beam models [142-144]. Wang and Feng [142] proposed a modified Euler model so as to examine the influence of surface elastic constants and surface residual stress on the buckling of nanowires subject to uniaxial compression. Wang [143] carried out a nonlinear analysis on the buckling of nanobeams conveying fluid flow; the nonlinear buckling of the fluid-conveying nanosystem was considerably affected by the surface elastic constant. More lately, the effects of surface energy on the

mechanics of non-homogeneous nanobeams have been analysed based on two different size-dependent theories; surface effects become more prominent as the material gradient index of non-homogeneous nanobeams increases.

It has been shown that the critical buckling load of different types of nanobeams such as CNTs and non-homogeneous nanobeams is sensitive to *temperature changes* [145-149]. Figure 15 shows temperature effects on the linear stability of CNTs. The temperature change affects the buckling force of SWCNTs. At low temperatures, the buckling force increases when the temperature change increases whereas increasing temperature change reduces the buckling force at a high temperature environment. This is because the thermal expansion constant of SWCNTs is negative at low temperatures while it is positive at high temperatures [150].

The *post-buckling* analysis of nanoscale beams has been the focus of many studies in the literature [151-153]. For example, Setoodeh et al. [154] determined exact analytical solutions for the post-buckling of SWCNTs within the framework of the nonlocal elasticity as well as the Euler–Bernoulli beam theory; the nonlinearity related to the stretching of the mid-plane is more profound for higher modes. In addition, the NSGT was utilised in order to examine the nonlinear buckling of nanoscale beams [155]; both the nonlocal and strain gradient parameters significantly affect the nonlinear buckling loads. The post-buckling of non-homogeneous nanobeams was also studied via the NSGT [156]; it was found that both hardening and softening responses can occur for the stiffness of the nanoscale beam depending on size coefficient values.

### 3.3.D. Size-dependent vibration of nanobeams

The nonlocal elasticity theory has been broadly utilised for analysing the *vibration characteristics* of nanobeams [157-164]. Some pioneering studies are briefly reviewed in the following. Wang and Varadan [165] proposed a linear nonlocal beam theory for the size-dependent oscillations of both single- and double-walled CNTs. The ratio of the nonlocal natural frequency to the local one decreases with increasing the nonlocal parameter as seen from Fig. 16 [166]. Murmu and Pradhan [167] analysed the oscillation of SWCNTs surrounded by a linear elastic medium employing the nonlocal elasticity incorporating thermal influences; the small scale influence becomes less important with increasing Winkler stiffness constant of the elastic medium (see Fig. 17). In addition, a nonlocal beam model was presented by Simsek [168] for the size-dependent vibration of CNTs subject to a moving load; the nonlocal dynamic deflection is larger than the local one since the small scale effect has a decreasing effect on the nanotube stiffness. Benzair et al. [169] examined temperature influences on the vibrations of CNTs via help of the nonlocal elasticity; temperature influences on the natural frequency decrease with increasing the vibration mode number. Duan et al. [170] employed the molecular dynamics to calibrate the nonlocal beam model of CNTs for the vibration analysis; it was found that the calibration coefficient of the nonlocal beam model depends on the geometrical features, mode number and edge conditions.

In addition to the stress nonlocality, *surface influences* on the vibration of nanoscale beams have been studied based on modified continuum models [75, 76, 171-175]; it was concluded that surface effects can account for the stiffness hardening, which cannot be described by the nonlocal elasticity theory. In addition, recently size-dependent continuum models incorporating

surface and nonlocal effects have been developed for the vibration of *smart* nanoscale beams such as piezoelectric and magneto-electro-elastic nanobeams [176-181] as well as *non-homogeneous* nanoscale beams [144, 182-185].

More recently, NSGT-based continuum models have been introduced for the vibration of nanobeams [82, 126, 183, 186-189]. Various modified theories of elasticity are compared in Fig. 18; CT, NT and SGT stand for the classical, nonlocal, and strain gradient theories, respectively. It is found that the NT gives the lowest natural frequencies while the SGT leads to the highest ones. The results of the NSGT are greatly dependent on the relative values of size parameters. For  $ea > l$ , the natural frequency obtained by the NSGT is higher than that of the NT but lower than that obtained by the CT. However, for  $ea < l$ , the NSGT leads to the natural frequency which is higher than that obtained by the CT but lower than that of the SGT.

In addition to linear modified continuum models, *nonlinear* size-dependent models have been presented in the literature for the free and forced vibrations of nanobeams using the nonlocal elasticity [182, 190-195], the surface elasticity [78, 196] and the NSGT [129, 183, 189]. Furthermore, different solution methods such as the differential quadrature method (DQM) [78, 190], the Homotopy perturbation method [182], the continuation scheme [189] and the Hamiltonian approach [183] have been utilised for solving the derived nonlinear equations of motion. Figure 19 shows the frequency-amplitude response of tubes at nanoscales via the NSGT;  $q_1$ ,  $\omega_1$  and  $\Omega$  denote the first generalised coordinate, the linear natural frequency and the non-dimensional excitation frequency, respectively. Perfecting straight nanotubes exhibit a hardening-type nonlinear response with two saddle nodes.

### 3.3.E. Size-dependent wave propagations in nanobeams

Wave propagations in nanobeams have been also analysed via help of size-dependent models including the nonlocal elasticity theory (NET) [197-200], the surface elasticity [201, 202] and the NSGT [59, 129, 203-205]. Figure 20 illustrates the change of the phase velocity with the wave number for various modified theories such as the classical elasticity theory (CET), NSGT, SGT and NET; the results of MD calculations are also plotted in the figure. The NSGT results are very close to those calculated by the MD simulations.

### 3.4. Nanoplates

Nanoplates such as graphene sheets [9], silver nanoplates [206] and metallic carbon nanosheets [207] have an extensive range of promising applications in various fields of nanotechnology. In applications such as nanomechanical resonators [208-211], nanoscale mass sensors [212-214] and actuators [215, 216], mechanical characteristics of nanoplates play an important role in the general performance of the nanoscale device. Figure 21 shows the application of graphene sheets as a resonating nanomechanical sensor [217]. So far many modified theoretical models have been reported on the mechanics of various nanoplates. In the following, first the linear motion equations for the mechanics of nanoplates are presented. Then, important size-dependent studies on the vibration, stability and static deformation of nanoscale plates as well as wave propagations in them are discussed.

#### 3.4.A. Nonlocal plate model

In Fig. 22, a typical single-layered graphene sheet (SLGS) with length  $l_x$  and width  $l_y$  is shown. Based on the NET, the constitutive equations of orthotropic nanoplates are expressed as

$$\begin{aligned}
\left[1 - (e_0 a)^2 \nabla^2\right] \sigma_{xx} &= \frac{E_1}{1 - \nu_{12} \nu_{21}} \varepsilon_{xx} + \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} \varepsilon_{yy}, \\
\left[1 - (e_0 a)^2 \nabla^2\right] \sigma_{yy} &= \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} \varepsilon_{xx} + \frac{E_2}{1 - \nu_{12} \nu_{21}} \varepsilon_{yy}, \\
\left[1 - (e_0 a)^2 \nabla^2\right] \sigma_{xy} &= 2G_{12} \varepsilon_{xy},
\end{aligned} \tag{22}$$

where  $E_i$ ,  $\nu_{ij}$  and  $G_{ij}$  stand for the elasticity modulus along the  $i$  direction, Poisson's ratio and the shear elasticity modulus of the nanoplate, respectively. The strain components of the orthotropic nanoplate are as

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} - z \frac{\partial^2 w}{\partial y^2}, \\
\varepsilon_{xy} &= \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - z \frac{\partial^2 w}{\partial x \partial y},
\end{aligned} \tag{23}$$

in which  $w$ ,  $v$  and  $u$ , respectively, indicate the mid-surface displacements in  $z$ ,  $y$  and  $x$  axes. The stress resultants are as

$$\begin{aligned}
\langle N_{xx}, N_{yy}, N_{xy} \rangle &= \int_{-h/2}^{h/2} \langle \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \rangle dz, \\
\langle M_{xx}, M_{yy}, M_{xy} \rangle &= \int_{-h/2}^{h/2} \langle \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \rangle z dz.
\end{aligned} \tag{24}$$

Here  $h$  denotes the thickness of the orthotropic nanoplate. Using Eqs. (22)-(24), the stress resultants of the orthotropic nanoplate are obtained as

$$\begin{aligned}
\left[1 - (e_0 a)^2 \nabla^2\right] N_{xx} &= S_{11} \frac{\partial u}{\partial x} + S_{12} \frac{\partial v}{\partial y}, \\
\left[1 - (e_0 a)^2 \nabla^2\right] N_{yy} &= S_{12} \frac{\partial u}{\partial x} + S_{22} \frac{\partial v}{\partial y},
\end{aligned}$$

$$\left[1 - (e_0 a)^2 \nabla^2\right] N_{xy} = S_{33} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad (25)$$

$$\left[1 - (e_0 a)^2 \nabla^2\right] M_{xx} = -D_{11} \frac{\partial^2 w}{\partial x^2} - D_{12} \frac{\partial^2 w}{\partial y^2},$$

$$\left[1 - (e_0 a)^2 \nabla^2\right] M_{yy} = -D_{12} \frac{\partial^2 w}{\partial x^2} - D_{22} \frac{\partial^2 w}{\partial y^2},$$

$$\left[1 - (e_0 a)^2 \nabla^2\right] M_{xy} = -2D_{33} \frac{\partial^2 w}{\partial x \partial y}, \quad (26)$$

where

$$\begin{aligned} S_{11} &= \frac{E_1 h}{1 - \nu_{12} \nu_{21}}, \quad S_{12} = \frac{\nu_{12} E_2 h}{1 - \nu_{12} \nu_{21}}, \quad S_{22} = \frac{E_2 h}{1 - \nu_{12} \nu_{21}}, \quad S_{33} = G_{12} h, \\ D_{11} &= \frac{E_1 h^3}{12(1 - \nu_{12} \nu_{21})}, \quad D_{12} = \frac{\nu_{12} E_2 h^3}{12(1 - \nu_{12} \nu_{21})}, \quad D_{22} = \frac{E_2 h^3}{12(1 - \nu_{12} \nu_{21})}, \quad D_{33} = \frac{G_{12} h^3}{12}, \end{aligned} \quad (27)$$

in which  $S_{ij}$  and  $D_{ij}$  stand for the in-plane and flexural stiffnesses of the nanoscale plate. Using Hamilton's law, the motion equations in terms of stress resultants are obtained as

$$\begin{aligned} \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= \rho h \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} &= \rho h \frac{\partial^2 v}{\partial t^2}, \\ \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + q &= \rho h \frac{\partial^2 w}{\partial t^2}, \end{aligned} \quad (28)$$

where  $q$  and  $\rho$  are the distributed transverse load and the nanoplate mass density, respectively.

Substituting Eq. (26) into Eq. (28), one obtains

$$\begin{aligned}
& -D_{11} \frac{\partial^4 w}{\partial x^4} - 2(D_{12} + 2D_{33}) \frac{\partial^4 w}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 w}{\partial y^4} \\
& + \left[ 1 - (e_0 a)^2 \nabla^2 \right] q + \left[ 1 - (e_0 a)^2 \nabla^2 \right] \frac{\partial}{\partial x} \left( N_{xy} \frac{\partial w}{\partial y} + N_{xx} \frac{\partial w}{\partial x} \right) \\
& + \left[ 1 - (e_0 a)^2 \nabla^2 \right] \frac{\partial}{\partial y} \left( N_{xy} \frac{\partial w}{\partial x} + N_{yy} \frac{\partial w}{\partial y} \right) = \left[ 1 - (e_0 a)^2 \nabla^2 \right] \rho h \frac{\partial^2 w}{\partial t^2}.
\end{aligned} \tag{29}$$

### 3.4.B. Size-dependent bending of nanoplates

Nonlocal continuum mechanics has been utilised to analyse the *static deformation* of nanoscale plates. For instance, Duan and Wang [218] obtained exact solutions for the axisymmetric static deformation of circular SLGSs subject to transverse loading by decoupling nonlocal equations for both clamped and simply-supported edges. In addition, an exact solution was presented by Yan et al. [121] for the bending of rectangular nanoplates via the NET. Aghababaei and Reddy [219] explored the static deformation of rectangular nanoplates incorporating size effects via use of the NET; a modified third-order theory of shear deformations was proposed for nanoplates. Huang et al. [220] determined the scale parameter for the bending of SLGSs using the molecular dynamics. Moreover, a nonlinear nonlocal plate model was proposed for the large deflection of monolayer graphene sheets [221] as well as bilayer graphene sheets [222, 223]. Surface influences on the size-dependent bending of nanoscale plates have been also examined [224-226]; a positive value of surface constant reduces the nanoplate deflection. In Fig. 23, the results of the nonlocal and local plate models as well as MD results for the bending of nanoplates are compared [121]. The results of the nonlocal plate model are in a very good agreement with those calculated by MD simulations.



### 3.4.C. Size-dependent buckling of nanoplates

NET-based models have been employed for the *linear buckling* of nanoplates in recent years due to the simplicity and capability of these models in the size-dependent analysis of structures at nanoscale levels. Various solution methods such as analytical solution techniques [227-233], the DQM [234-236], Galerkin's approach [237, 238] and the finite strip method [239] have been applied to the nonlocal governing differential equations of nanoscale plates. Figure 24 shows the nonlocal-to-local buckling load ratio versus the scale parameter for nanoplates subject to uniaxial and biaxial loading conditions; the nonlocal buckling load is lower than the local one. This is because the nonlocal effect reduces the structural stiffness of nanoplates, and thus the buckling load declines. The accuracy and reliability of the nonlocal continuum modelling of nanoplates have been shown by performing MD simulations [87, 240]. In Fig. 25, the buckling force of square SLGSs versus the length is plotted for different size parameters (i.e.  $\mu = (e_0 a)^2$ ); the MD results are also shown. It is observed that the nonlocal plate model with a reasonable size parameter can accurately predict the critical buckling force of nanoscale plates.

In addition to the NET, the surface elasticity theory [87, 241-243] and the NSGT [244] have been also utilised for the stability of nanoscale plates, especially GSs. It has been shown that surface or strain gradient effects should be taken into consideration to capture the stiffness hardening behaviour observed in the stability of circular GSs subject to axisymmetric radial loads. Furthermore, nonlinear nonlocal models [245-247] and nonlinear continuum models incorporating surface effects [248, 249] have been developed in the literature to analyse the size-dependent post-buckling of nanoplates.

#### 3.4.D. Size-dependent vibration of nanoplates

Various nonlocal plate models such as the Kirchhoff plate theory [250-254], first-order shear deformation model [55, 255], two-variable refined theory of plates [256, 257] and higher-order shear deformation model [258-260] have been employed so as to examine the *linear vibration* of nanoscale plates. On the other hand, to solve the size-dependent differential equations of these nonlocal plate models, different solution methods such as analytical approaches [261-263], Galerkin's method [264, 265], the DQM [254, 266, 267], the finite element method [268, 269] and the kp-Ritz method [270, 271]. The variation of the fundamental frequency of square SLGSs with the width for various nonlocal parameters is illustrated in Fig. 26. The results of MD simulations, local and nonlocal plate models are given. Firstly, the results of the NET match MD results while the CET leads to overestimated results especially for SLGSs with small widths. Secondly, increasing nonlocal parameter reduces the fundamental frequency since increasing nonlocal effects reduces the structural stiffness. Moreover, it is found that the nonlocal influence gradually disappears as the width of the nanoplate increases.

The influences of *surface energy*, *residual surface tension* and *strain gradients* affect the vibration characteristics of nanoplates. In recent years, size-dependent plate models incorporating surface effects [80, 272-274] as well as NSGT-based models [275, 276] have been developed for the vibration of nanoplates. It has been shown that the stiffness-hardening behaviour can be described using these size-dependent plate models.

In addition to linear size-dependent plate models, *nonlinear* models have been proposed to analyse the large-amplitude vibration of nanoscale plates using the surface elasticity theory [277,

278] and the NET [279-282]. Figure 27 indicates the size influence on the nonlinear vibration of SLGSs with four edges simply supported. The nonlinear frequency ratio is defined as  $\omega/\omega_L$  where  $\omega$  and  $\omega_L$  indicate the nonlinear and linear nonlocal frequencies, respectively. It is found that the as the scale parameter increases the influence of the geometrical nonlinearity. More recently, size-dependent nonlinear plate models have been utilised for investigating the large-amplitude vibration of smart nanoscale plates such as piezoelectric [211, 283, 284] and magneto-electro-elastic ultrathin plates [285, 286].

#### 3.4.E. *Size-dependent wave propagations in nanoplates*

*Wave propagations* in nanoscale plates such as graphene sheets [287, 288], smart [289] and inhomogeneous [290] nanoplates have been examined using size-dependent plate models. The majority of size-dependent studies on the wave propagation analysis have been carried out via use of the NET [291-293]. The surface elasticity theory [294, 295] and the NSGT [296, 297] have been also utilised to explore the size-dependent wave propagation in nanoplates. It was found that increasing nonlocal parameter strengthens the dispersion degree. In addition, strengthened dispersion degrees for nanoplates can be achieved by increasing wave numbers.

#### 3.5. *Nanoshells*

In addition to size-dependent beam models, the mechanical behaviour of nanotubes including CNTs [3], boron nitride nanoscale tubes (BNNTs) [298] and piezoelectric nanoscale tubes (PNTs) [299] can be estimated via size-dependent shell models. In general, modified shell theories result in a more accurate estimation of the mechanical characteristics of nanotubes compared to modified beam theories. However, size-dependent shell models are more complex

in terms of mathematical formulation as well as solution methods. Moreover, they require high computational costs compared to size-dependent beam models.

### 3.5.A. Nonlocal shell model

At nanoscale levels, the NET is usually employed in order to incorporate the size effect into a continuum-based shell model. Based on the NET and the classical shell theory, one can write

$$\left[1 - (e_0 a)^2 \nabla^2\right] \begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{\theta z} \\ \sigma_{xz} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \gamma_{x\theta} \end{Bmatrix}, \quad (30)$$

where  $C_{ij}$  are the elasticity constants of the nanoscale shell. From the above equation, the normal strain along the z direction is obtained as  $\varepsilon_{zz} = -(1/C_{33})(C_{13}\varepsilon_{xx} + C_{23}\varepsilon_{\theta\theta})$ . Substituting this relation into Eq. (30), one obtains

$$\left[1 - (e_0 a)^2 \nabla^2\right] \begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & 0 \\ \tilde{C}_{12} & \tilde{C}_{22} & 0 \\ 0 & 0 & \tilde{C}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix}, \quad (31)$$

where

$$\tilde{C}_{11} = C_{11} - \frac{C_{13}^2}{C_{33}}, \quad \tilde{C}_{22} = C_{22} - \frac{C_{23}^2}{C_{33}}, \quad \tilde{C}_{12} = C_{12} - \frac{C_{13}C_{23}}{C_{33}}, \quad \tilde{C}_{66} = C_{66}. \quad (32)$$

Applying Hamilton's law, the motion equations of the nanoscale shell in terms of stress resultants are obtained as

$$\frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} + \frac{\partial N_{xx}}{\partial x} = \rho h \frac{\partial^2 u}{\partial t^2}, \quad (33)$$

$$\frac{1}{R} \frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \left( \frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial M_{\theta\theta}}{\partial \theta} \right) = \rho h \frac{\partial^2 v}{\partial t^2}, \quad (34)$$

$$\begin{aligned} & \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{1}{R} \left( \frac{1}{R} \frac{\partial^2 M_{\theta\theta}}{\partial \theta^2} + 2 \frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} - N_{\theta\theta} \right) + q \\ & + N_{xx}^t \frac{\partial^2 w}{\partial x^2} + \frac{N_{\theta\theta}^t}{R^2} \frac{\partial^2 w}{\partial \theta^2} = \rho h \frac{\partial^2 w}{\partial t^2}, \end{aligned} \quad (35)$$

where

$$\begin{Bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{Bmatrix} z dz, \quad (36)$$

in which  $h$  and  $\rho$  indicate the thickness and mass density of the nanoscale shell, respectively;

also,  $N_{\theta\theta}^t$  and  $N_{xx}^t$  are respectively the total circumferential and axial loads.  $R$  and  $q$  represent

the average radius of the nanoshell and the radial loading, respectively. Using Eqs. (31) and (36),

one can obtain

$$\begin{aligned} \left[ 1 - (e_0 a)^2 \nabla^2 \right] \begin{Bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \end{Bmatrix} &= \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} & 0 \\ \tilde{S}_{12} & \tilde{S}_{22} & 0 \\ 0 & 0 & \tilde{S}_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{1}{R} \left( \frac{\partial v}{\partial \theta} + w \right) \\ \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} \end{Bmatrix} + \begin{Bmatrix} N_{xx}^t \\ N_{\theta\theta}^t \\ 0 \end{Bmatrix}, \\ \left[ 1 - (e_0 a)^2 \nabla^2 \right] \begin{Bmatrix} M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \end{Bmatrix} &= - \begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{12} & 0 \\ \tilde{D}_{12} & \tilde{D}_{22} & 0 \\ 0 & 0 & \tilde{D}_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{1}{R^2} \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta} \right) \\ \frac{1}{R} \left( 2 \frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x} \right) \end{Bmatrix}, \end{aligned} \quad (37)$$

where

$$\begin{aligned}\langle \tilde{S}_{11}, \tilde{S}_{12}, \tilde{S}_{22}, \tilde{S}_{66} \rangle &= \langle \tilde{C}_{11}, \tilde{C}_{12}, \tilde{C}_{22}, \tilde{C}_{66} \rangle h, \\ \langle \tilde{D}_{11}, \tilde{D}_{12}, \tilde{D}_{22}, \tilde{D}_{66} \rangle &= \langle \tilde{C}_{11}, \tilde{C}_{12}, \tilde{C}_{22}, \tilde{C}_{66} \rangle \frac{h^3}{12}.\end{aligned}\quad (38)$$

Substituting Eq. (37) into Eqs. (33)-(35), the differential motion equations of the nanoshell are derived as

$$\tilde{S}_{11} \frac{\partial^2 u}{\partial x^2} + \frac{\tilde{S}_{66}}{R^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{R} (\tilde{S}_{12} + \tilde{S}_{66}) \frac{\partial^2 v}{\partial x \partial \theta} + \frac{\tilde{S}_{12}}{R} \frac{\partial w}{\partial x} = \tilde{m} [1 - (e_0 a)^2 \nabla^2] \frac{\partial^2 u}{\partial t^2}, \quad (39)$$

$$\begin{aligned}\frac{1}{R} (\tilde{S}_{12} + \tilde{S}_{66}) \frac{\partial^2 u}{\partial x \partial \theta} + \left( \tilde{S}_{66} + \frac{\tilde{D}_{66}}{R^2} \right) \frac{\partial^2 v}{\partial x^2} + \frac{1}{R^2} \left( \tilde{S}_{22} + \frac{\tilde{D}_{22}}{R^2} \right) \frac{\partial^2 v}{\partial \theta^2} \\ + \frac{1}{R^2} \left\{ \tilde{S}_{22} \frac{\partial w}{\partial \theta} - \left[ \frac{\tilde{D}_{22}}{R^2} \frac{\partial^3 w}{\partial \theta^3} + (\tilde{D}_{12} + 2\tilde{D}_{66}) \frac{\partial^3 w}{\partial x^2 \partial \theta} \right] \right\} = \tilde{m} [1 - (e_0 a)^2 \nabla^2] \frac{\partial^2 v}{\partial t^2},\end{aligned}\quad (40)$$

$$\begin{aligned}-\frac{1}{R} \left( \tilde{S}_{12} \frac{\partial u}{\partial x} + \frac{\tilde{S}_{22}}{R} \frac{\partial v}{\partial \theta} \right) + \frac{1}{R^2} \left[ (\tilde{D}_{12} + 2\tilde{D}_{66}) \frac{\partial^3 v}{\partial x^2 \partial \theta} + \frac{\tilde{D}_{22}}{R^2} \frac{\partial^3 v}{\partial \theta^3} \right] \\ - \left\{ \tilde{D}_{11} \frac{\partial^4 w}{\partial x^4} + \frac{1}{R^2} \left[ 2(\tilde{D}_{12} + 2\tilde{D}_{66}) \frac{\partial^4 w}{\partial x^2 \partial \theta^2} + \frac{\tilde{D}_{22}}{R^2} \frac{\partial^4 w}{\partial \theta^4} + \tilde{S}_{22} w \right] \right\} \\ + [1 - (e_0 a)^2 \nabla^2] \left( q + \frac{N_{\theta\theta}^t}{R^2} \frac{\partial^2 w}{\partial \theta^2} + N_{xx}^t \frac{\partial^2 w}{\partial x^2} \right) = \tilde{m} [1 - (e_0 a)^2 \nabla^2] \frac{\partial^2 w}{\partial t^2},\end{aligned}\quad (41)$$

in which  $\tilde{m} = \rho h$ . The above coupled differential equations govern the size-dependent mechanical behaviour of nanoshells based on the NET.

### 3.5.B. Size-dependent mechanics of nanoshells

Size-dependent shell models have been recently applied for analysing the mechanics of CNTs [300-302], BNNTs [303], piezoelectric nanotubes [304, 305] and magneto-electro-elastic nanotubes [306, 307]. To modify the traditional shell theories for capturing size influences,

various modified theories including the NET [300, 301, 308], the surface elasticity [309, 310] and the NSGT [311, 312] can be used. The NET can predict the stiffness softening while the surface elasticity with positive surface properties can account for the stiffness hardening. The NSGT takes into account both the softening and hardening of the structural stiffness. To solve the motion equations of nanotubes using modified shell models, various solution techniques such as the DQM [313, 314], perturbation schemes [311, 315], analytical methods [308, 316, 317] were employed.

The appropriate value of the nonlocal parameter has a significant role in the correct prediction of the mechanical characteristics of nanotubes, especially CNTs. Wang and Wang [318] formulated the constitutive equations of CNTs via use of the nonlocal elasticity; they also proposed a general range for the nonlocal parameter (i.e.  $0 \text{ nm} \leq e_0 a \leq 2 \text{ nm}$ ), which has been extensively applied to the nonlocal continuum models of CNTs and GSs. Particularly, Ansari et al. [319] calibrated a nonlocal shear deformable shell model for the buckling of SWCNTs applying the MD. The value of the nonlocal parameter ranges from 0.5 to 0.8 nm depending on the bending rigidity and the boundary conditions. Figures 28 and 29 show the change of the buckling force with  $L/d$  for both local and nonlocal shell models for clamped-free and simply-supported edges. The local shell model fails to correctly predict the buckling behaviour of SWCNTs while the nonlocal shell model with a calibrated scale parameter gives a reasonable estimation of the critical buckling force. Tables 1 and 2 list the calibrated scale parameters of CNTs and GSs for the NET, respectively. In addition, the calibrated values for the scale parameters of CNTs for the NSGT are given in Table 3.

#### 4. Conclusions

Nanostructures have been extensively utilised in NEMS-based devices including nanomechanical resonators, mass nanosensors, nanoenergy harvesters, and nanogenerators owing to their excellent physical characteristics. To properly design and manufacture these small-scale devices, it is important to estimate how nanostructures respond to mechanical/electrical excitation loads. Due to the problems associated with performing experiments at nanoscales and conducting MD simulations, the size-dependent continuum models of nanostructures have received a considerable attention.

The NET and NSGT have widely been applied for estimating the mechanical characteristics of nanoscale structures. In the present paper, first these size-dependent elasticity theories were briefly reviewed. Then, the NET-based equations of motion were derived for nanorods, nanorings, nanobeams, nanoplates and nanoshells. Pioneering studies conducted on the size-dependent continuum modelling of nanostructures with and without incorporating surface effects on the basis of the NET and NSGT were reviewed. Important findings on the mechanical behaviour of these nanostructures are summarised as

- Scale effects predicted by the NSGT appear in a wider range of lengths compared to nonlocal effects.
- The computational cost of the NET is less than that of the NSGT.
- Surface effects are more significant for slender nanobeams.
- Increasing strain gradient parameter reduces the transverse deflection of nanobeams while the transverse deflection increases with increasing nonlocal coefficient.



- For cantilever nanobeams, integral nonlocal models can better describe the bending behaviour than a differential nonlocal model.
- Nonlocal effects reduce the critical buckling loads of nanostructures due to the reduction of the structural stiffness.
- Increasing nonlocal parameter notably reduces the natural frequencies of nanostructure.
- Strengthened dispersion degrees for nanoplates can be achieved by increasing the nonlocal parameter as well as increasing the wave number.
- For circular nanoplates, a combination of the NET and surface elasticity or the NSGT is required in order to accurately predict the critical buckling loads.
- Size-dependent shell models lead to a more accurate estimation of the mechanical characteristics of nanotubes compared to size-dependent beam models.
- Size-dependent shell models are more mathematically complex and require high computational effort compared to size-dependent beam models.

Further effort is required in order to calibrate the available size-dependent continuum models of nanostructures by performing MD simulations or experiments. In addition, compared to nanobeams, the mechanical behaviour of nanorings has not been studied comprehensively. More analysis can be carried out to explore the size-dependent mechanical response of nanorings, especially in thermal environment. Moreover, most size-dependent studies on the mechanics of nanorods are linear. Modified continuum models can be developed to examine the nonlinear vibration of nanorods.

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Table 1: Calibrated scale parameters for the mechanics of CNTs via the NET.

Mechanical behaviour	Continuum model	Calibration parameter ( $e_0$ )	Nonlocal parameter (nm)
Mechanics	NET-based shell and beam models [318]	----	0-2
Wave propagation	NET-based shell model [301]	0.2-0.6	----
Vibration	NET-based beam model [170]	0-19	----
Wave propagation	NET-based rod model [101]	0-0.386	----
Axial buckling	NET-based shell model [319]	----	0.531-0.780

Table 2: Calibrated scale parameters for the mechanics of GSs via the NET.

Mechanical behaviour	Continuum model	Nonlocal parameter (nm)
Buckling	NET-based plate model [240]	1.33-1.36
Vibration	NET-based plate model [279]	0.22-0.67
Post-buckling	NET-based nonlinear plate model [246]	0.25-1

Table 3: Calibrated scale parameters for the mechanics of CNTs via the NSGT.

Mechanical behaviour	Continuum model	Strain gradient parameter (nm)	Nonlocal parameter (nm)
Buckling	NSGT-based shell model [320]	0.4-0.9	1-1.5
Vibration	NSGT-based shell model [321]	0.1-0.4	3.3-3.5
Wave propagation	NSGT-based beam model [129]	0.175	0.8

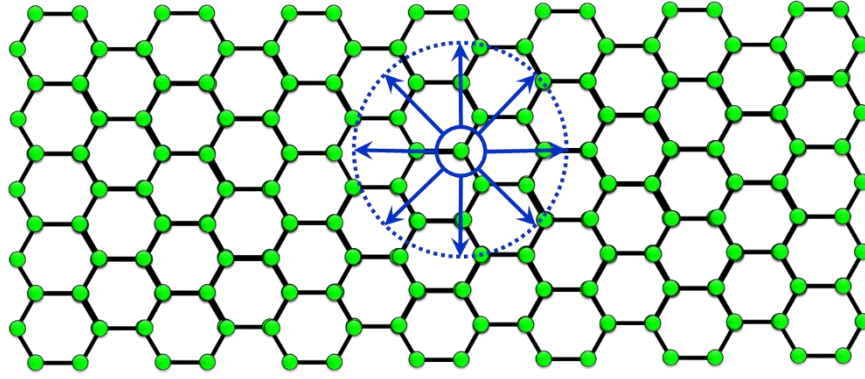


Fig. 1. Stress at a spot of a nanostructure is dependent on strains at all spots according to the nonlocal elasticity theory.

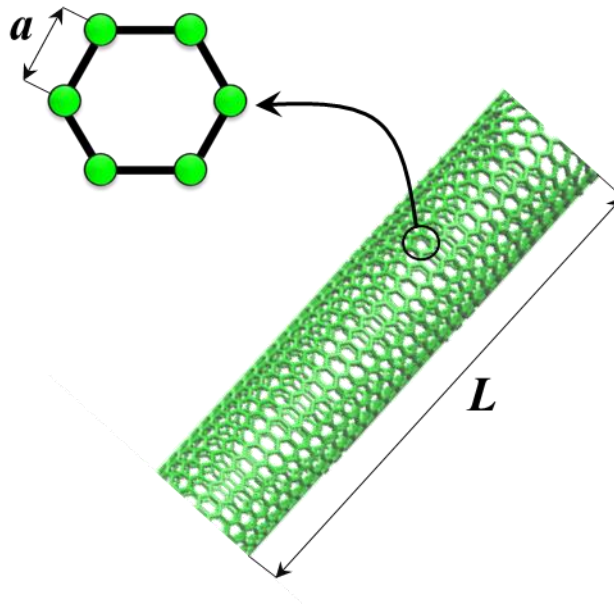


Fig. 2. Internal characteristics length ( $a$ ) as well as the external characteristics length ( $L$ ) for CNTs.

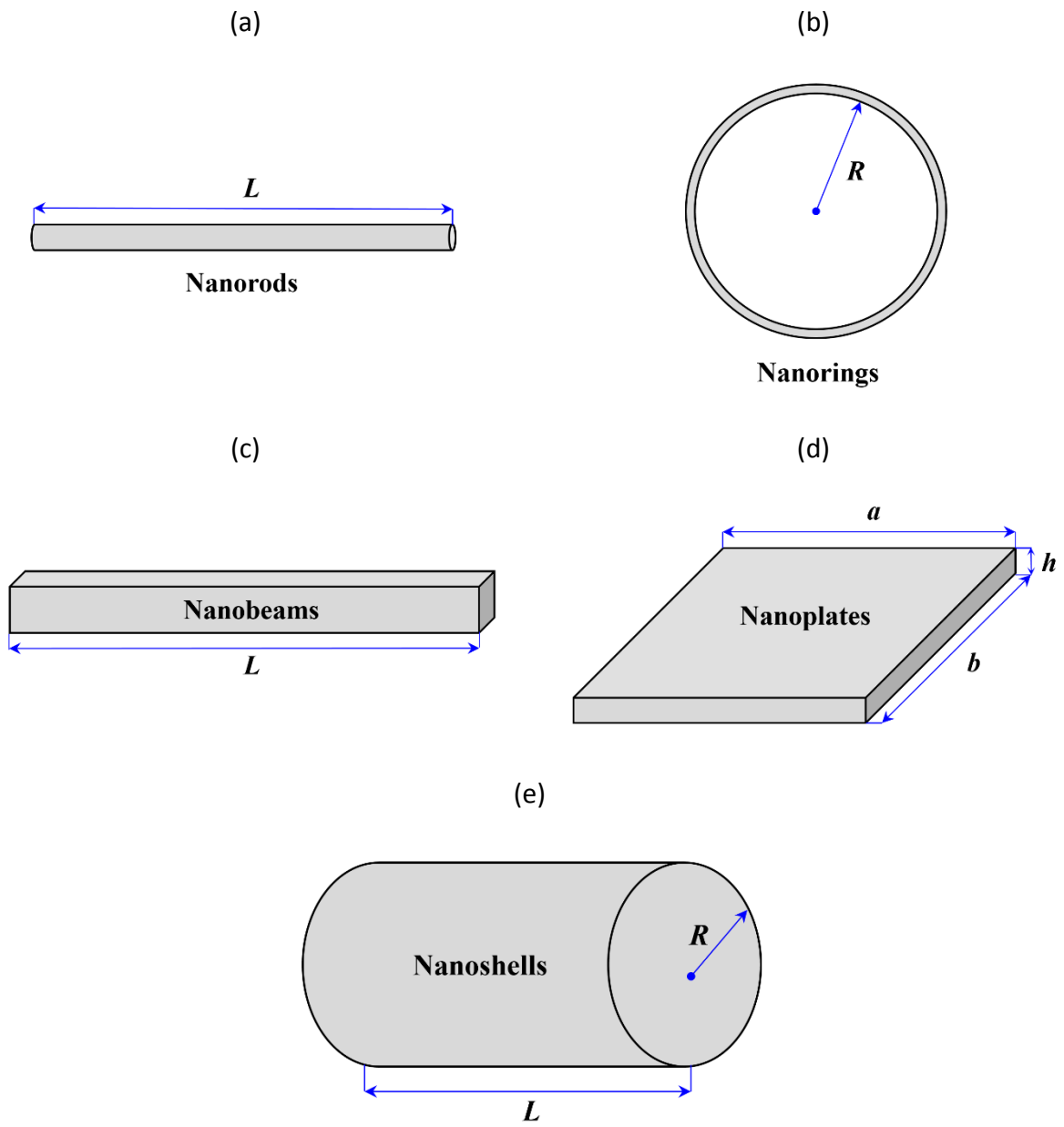


Fig. 3. Different types of nanostructures: a) nanorods, b) nanorings, c) nanobeams, d) nanoplates, and e) nanoshells.

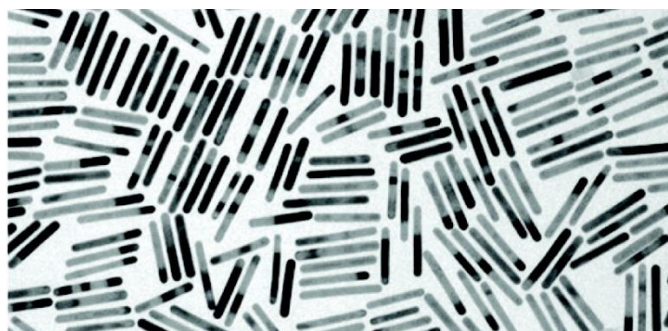


Fig. 4. A set of gold nanorods [88]. Reprinted with permission from ACS. Copyright (2012) American Chemical Society.

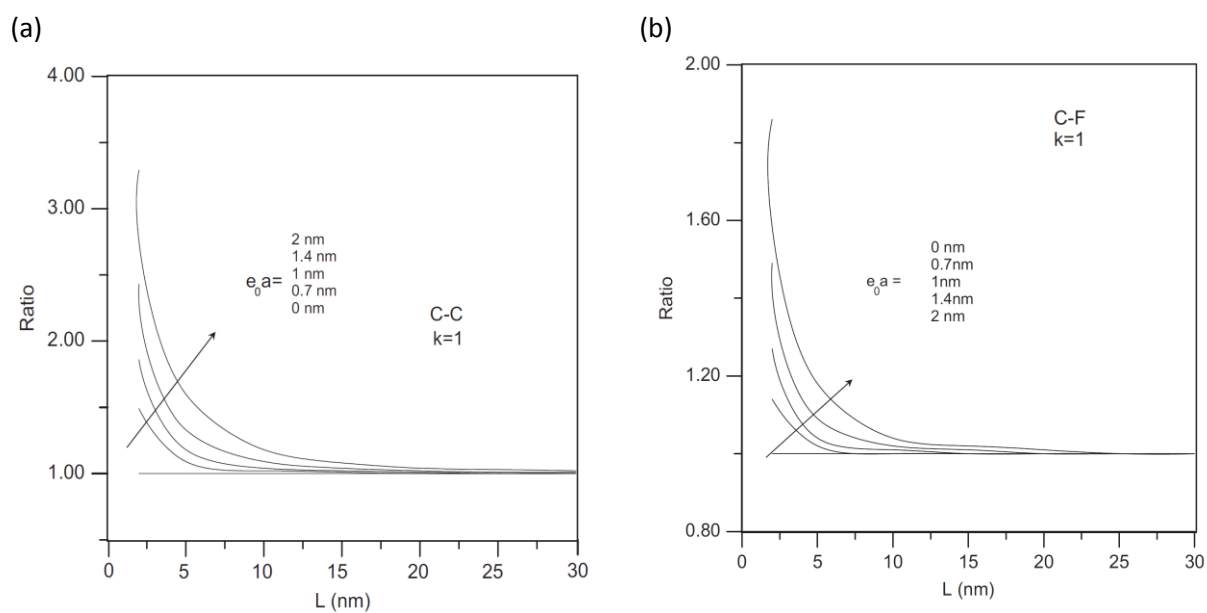


Fig. 5. Scale effects on the longitudinal vibration of nanorods for (a) C-C and (b) C-F ends [74]. Reprinted with permission from Elsevier.

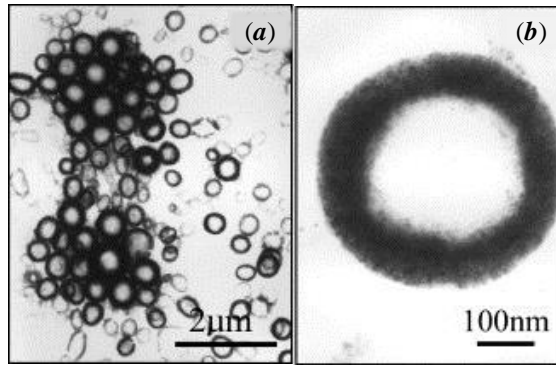


Fig. 6. (a) A system of nanorings and (b) a single nanoring [2]. Reprinted with permission from Elsevier.

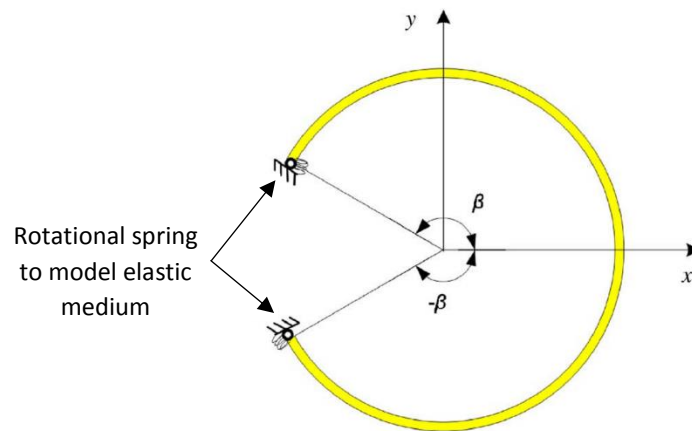


Fig. 7. A nanoscale arch embedded in an elastic medium [105]. Reprinted with permission from AIP Publishing.

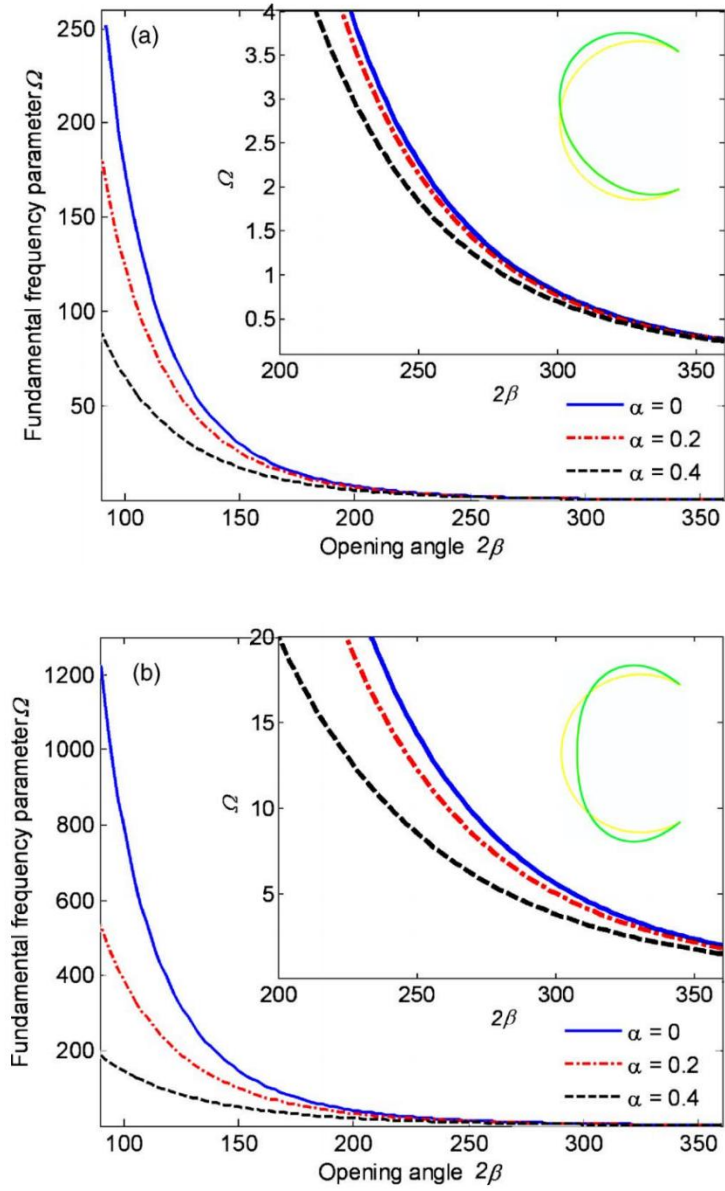


Fig. 8. Frequency parameter versus the opening angle for various nonlocal parameters for (a) asymmetric and (b) symmetric modes [105]. Reprinted with permission from AIP Publishing.

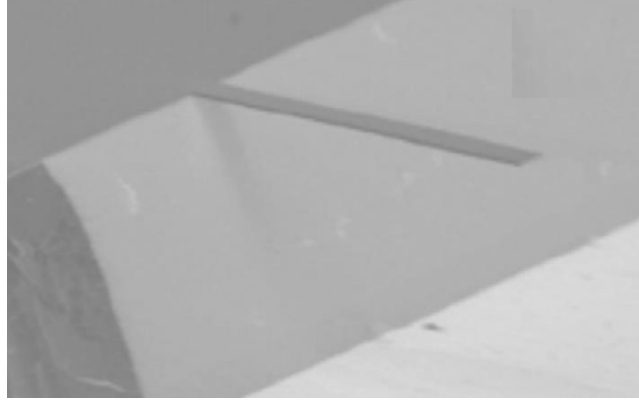


Fig. 9. A schematic representation of a cantilever nanobeam [110]. Reprinted with permission from AIP Publishing.

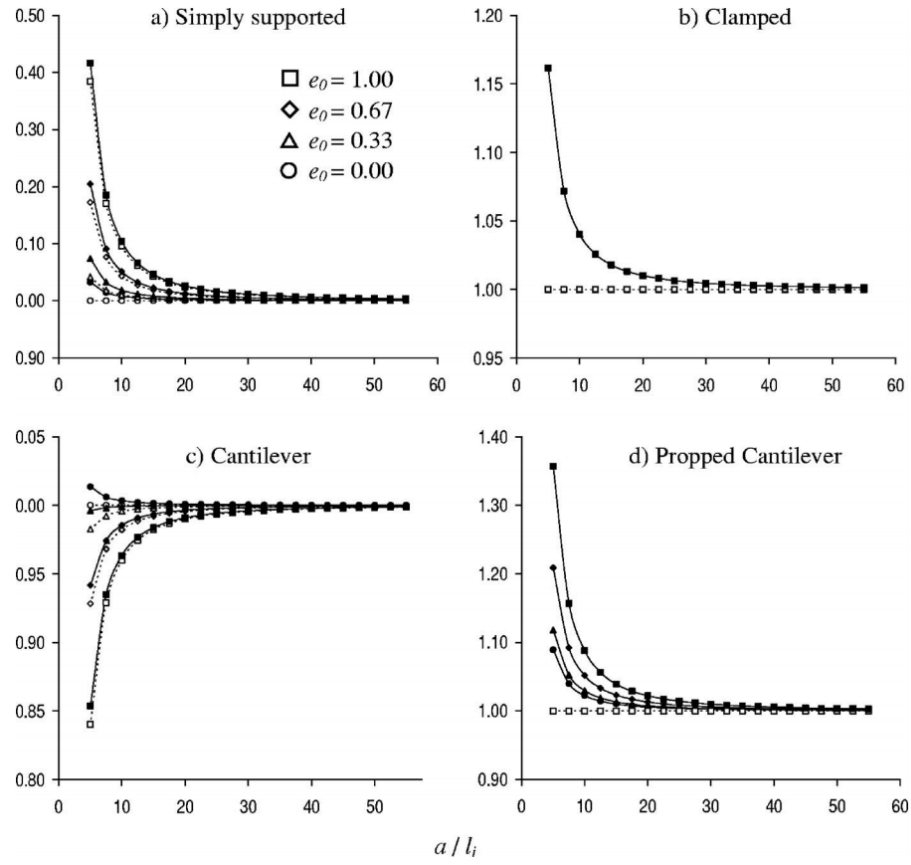


Fig. 10. The ratio of the maximum transverse displacement calculated via the Timoshenko model of beams to that calculated via the Euler-Bernoulli model against the ratio of the length to the internal characteristic length for (a) simply supported, (b) clamped, (c) cantilever and (d) propped cantilever nanobeams of nanobeams subject to a uniform transverse load (a dashed line indicates the Euler-Bernoulli model while a solid line indicates Timoshenko beam model) [117]. Reprinted with permission from AIP Publishing.



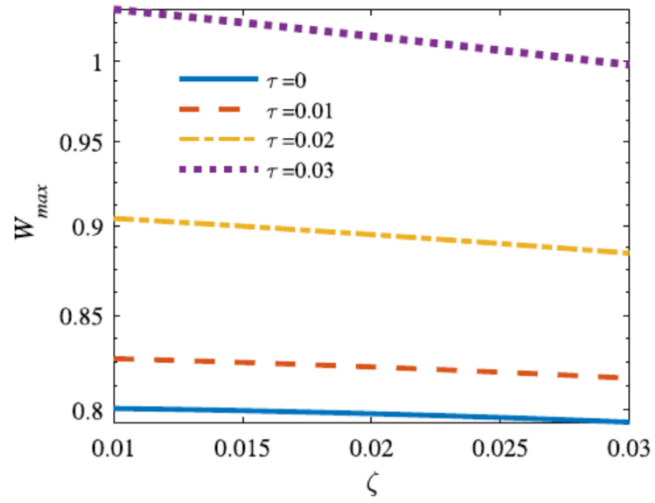


Fig. 11. Maximum deflection of a non-homogeneous nanoscale beam with simply supported boundary conditions subject to sinusoidal applied load for different strain gradient coefficients and various nonlocal coefficients [126]. Reprinted with permission from Elsevier.

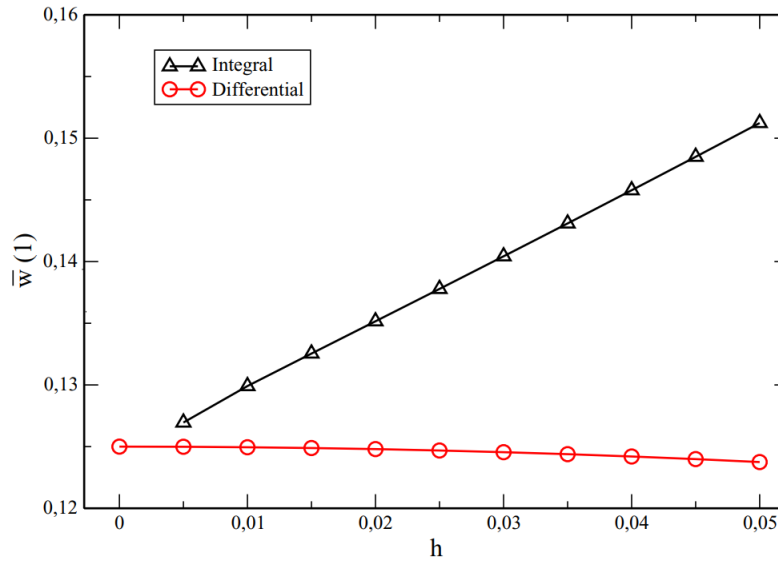


Fig. 12. Integral and differential nonlocal models for the bending of cantilever nanobeams ( $h$  is the nonlocal coefficient defined as  $h=e_0a/L$ ) [130]. Reprinted with permission from Elsevier.

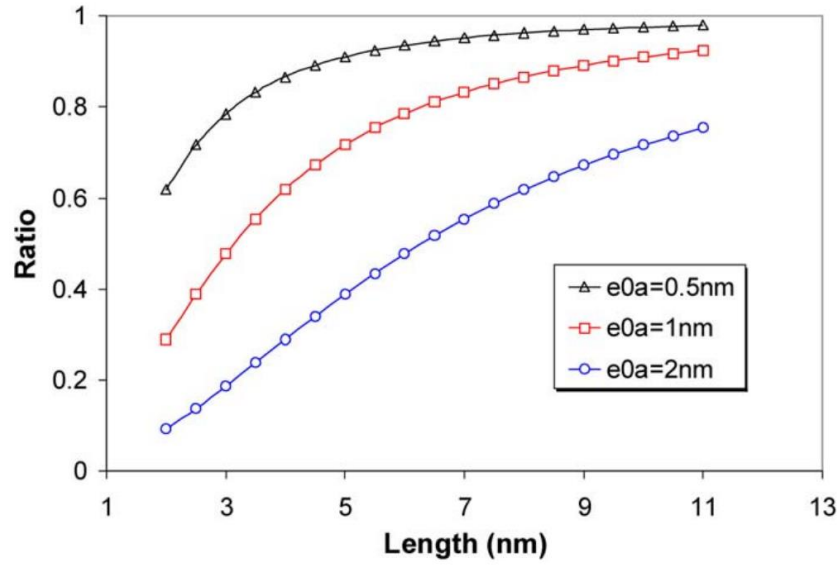


Fig. 13. The nonlocal-to-local buckling ratio of CNTs against their length for various  $e_0a$  [141]. Reprinted with permission from Elsevier.

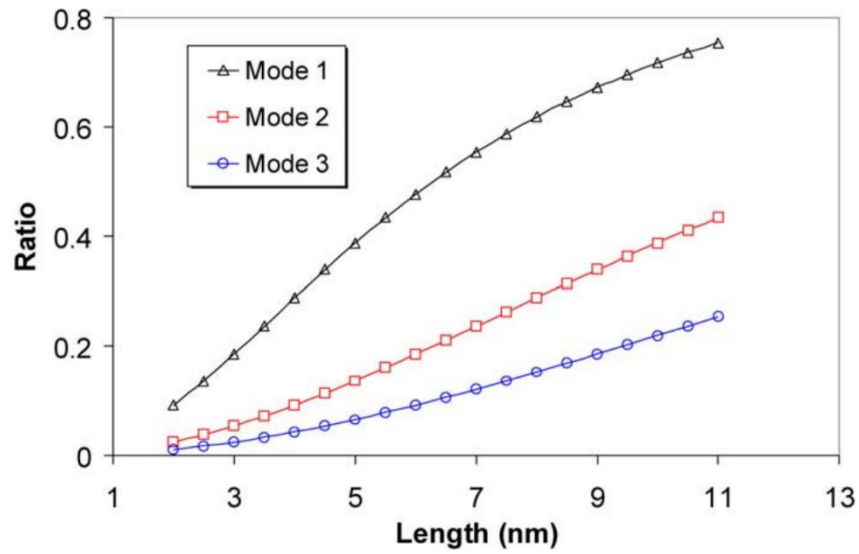


Fig. 14. The nonlocal-to-local buckling ratio of CNTs against their length for different mode numbers [141]. Reprinted with permission from Elsevier.

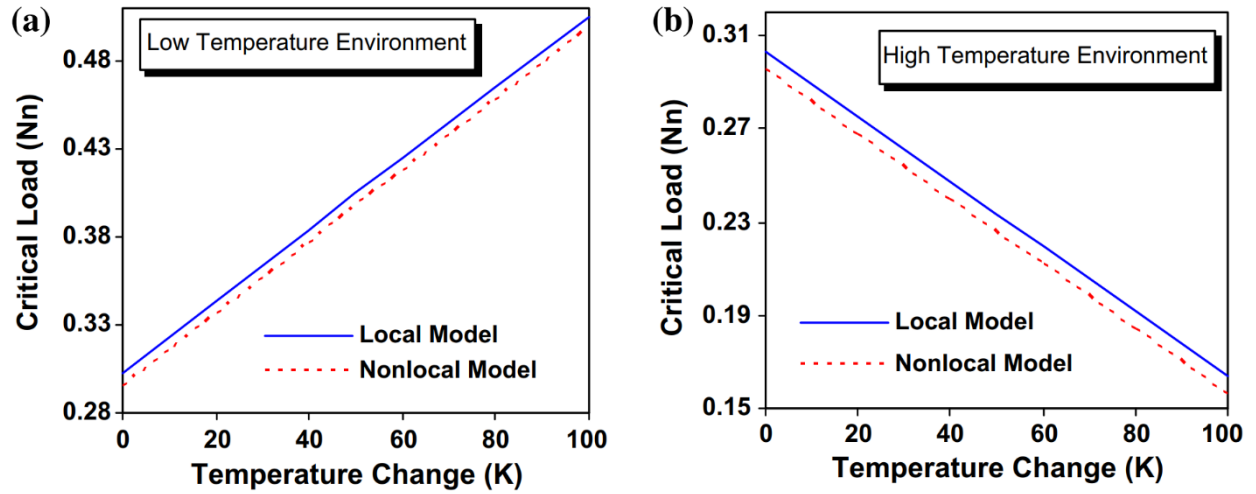


Fig. 15. The critical buckling load versus the temperature change for (a) low temperature environment, and (b) for high temperature environment [145]. Reprinted with permission from Elsevier.

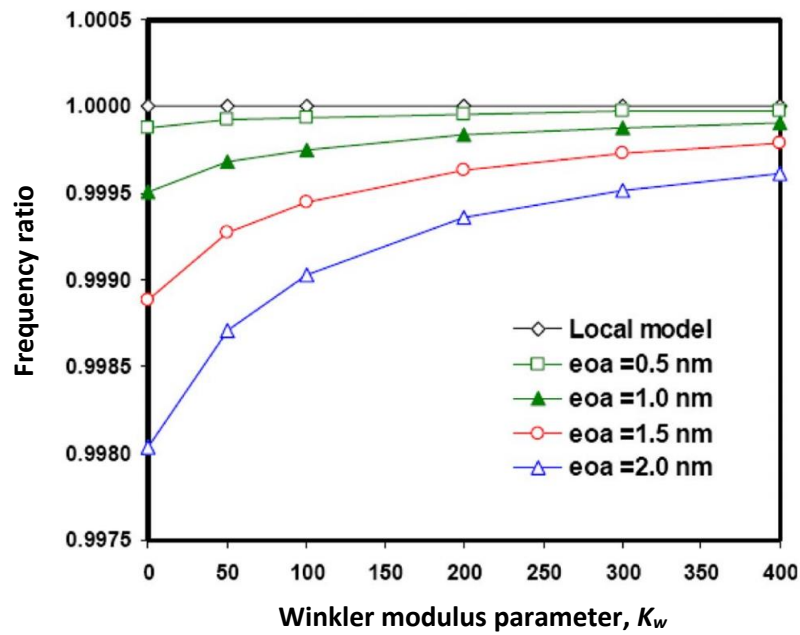


Fig. 16. The ratio of the nonlocal frequency to the local one versus the Winkler parameter for various nonlocal coefficients [166]. Reprinted with permission from AIP Publishing.

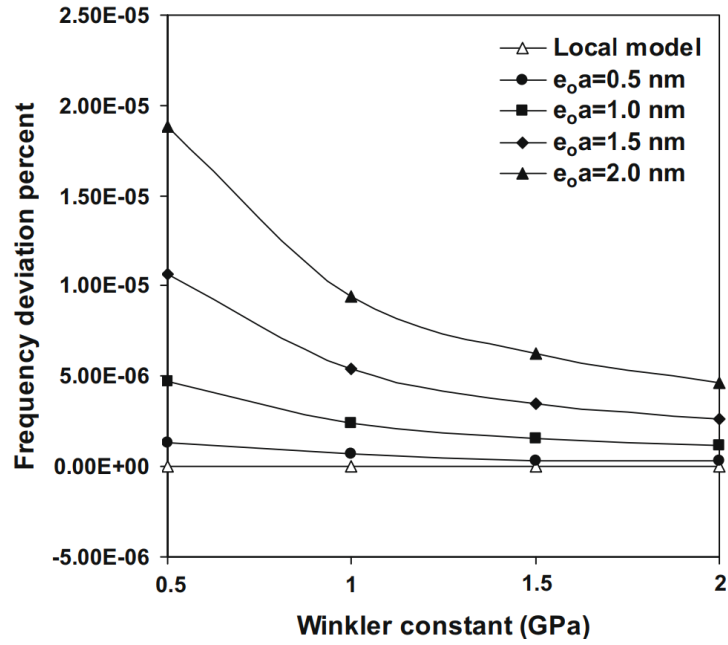


Fig. 17. The frequency difference between the local and nonlocal models versus the Winkler constant for different nonlocal parameters [167]. Reprinted with permission from Elsevier.

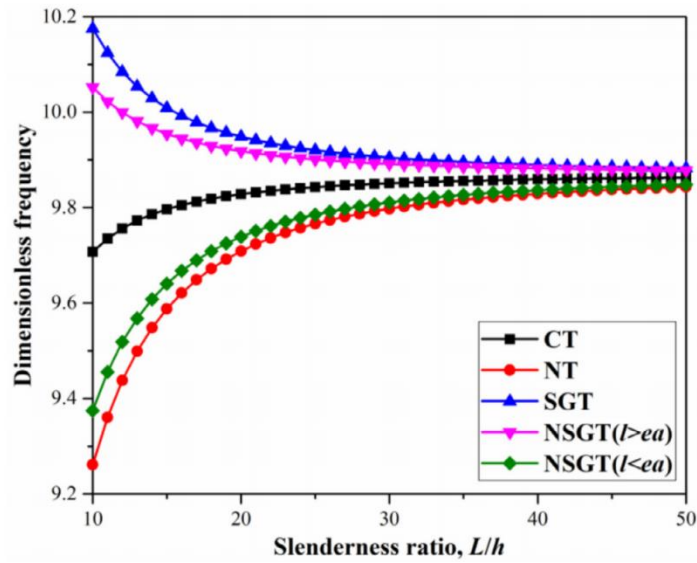


Fig. 18. Dimensionless frequency versus the slenderness ratio of nanoscale beams for various elasticity theories [186]. Reprinted with permission from Elsevier.

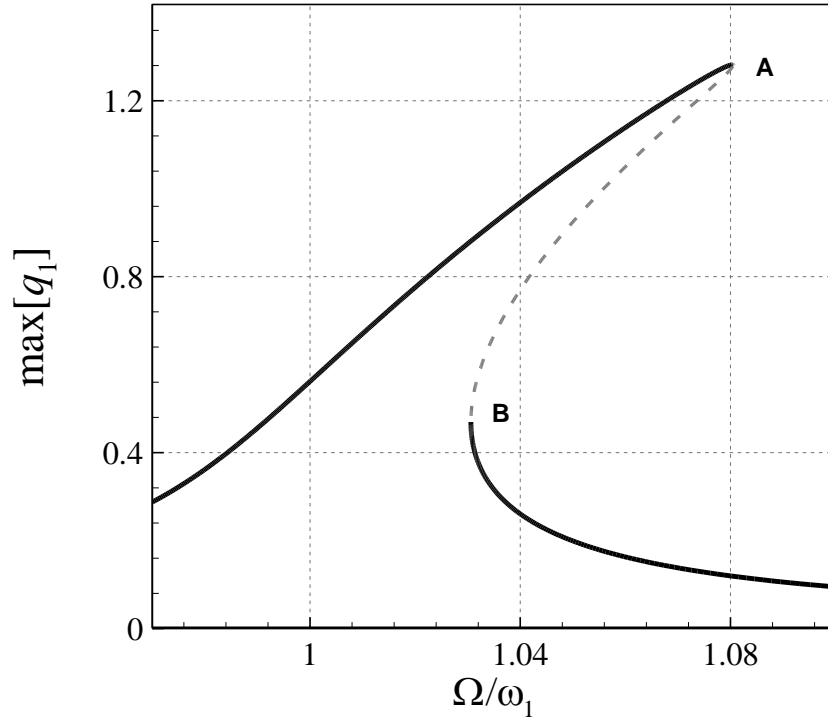


Fig. 19. Frequency-amplitude response of tubes at nanoscales via the NSGT [189]. Reprinted with permission from Elsevier.

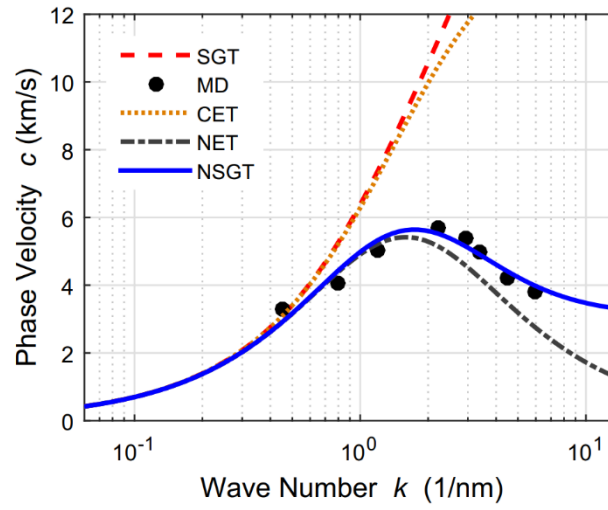


Fig. 20. Wave propagations in SWCNTs based on different modified theories [129]. Reprinted with permission from Elsevier.

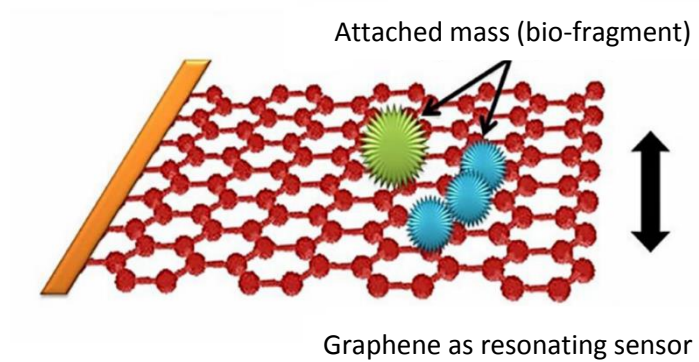
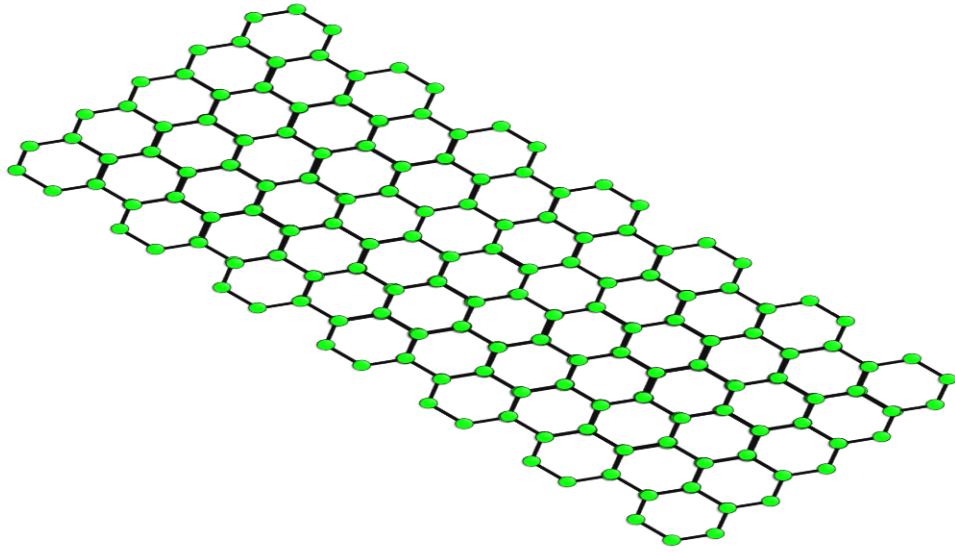


Fig. 21. Application of graphene sheets as a resonating nanosensor [217]. Reprinted with permission from Elsevier.

(a)



(b)

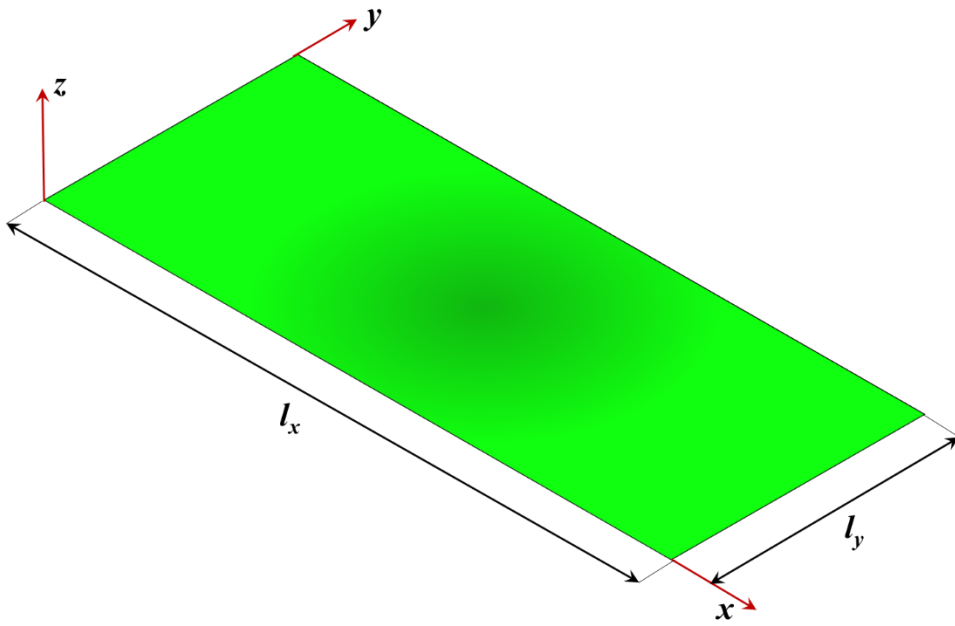


Fig. 22. (a) A SLGS with length  $l_x$  and width  $l_y$ , and (b) its equivalent continuum model.

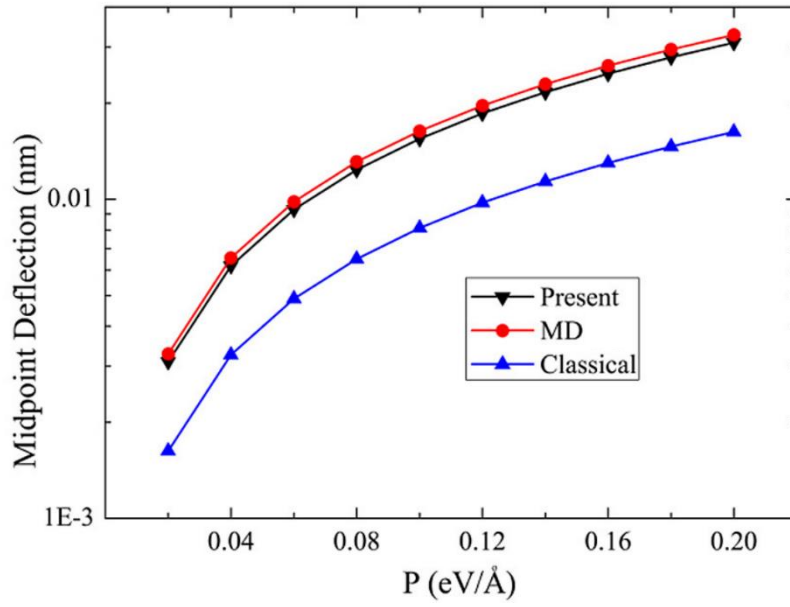


Fig. 23. Comparison of the nonlocal and local plate models as well as MD results [121]. Reprinted with permission from Elsevier.

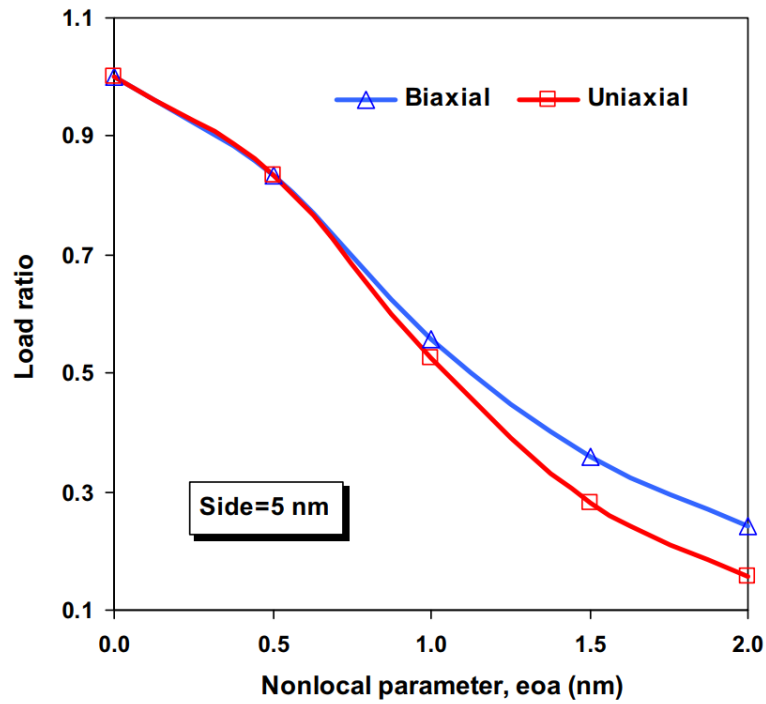


Fig. 24. Buckling load ratio versus the nonlocal parameter for both biaxially and uniaxially compressed nanoplates [235]. Reprinted with permission from Elsevier.



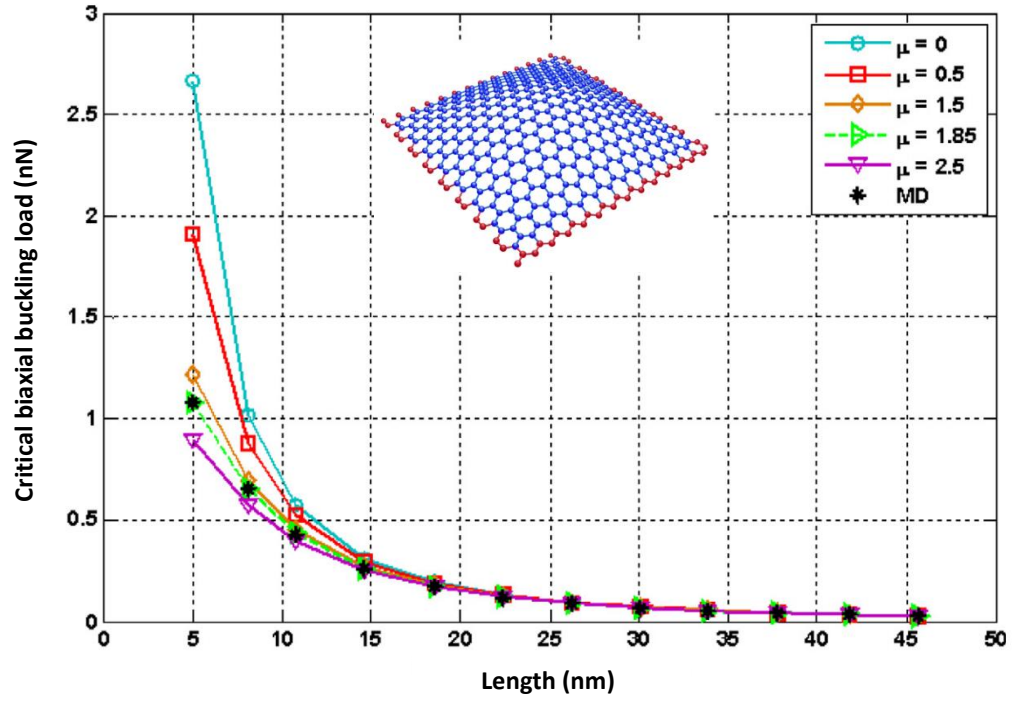


Fig. 25. Accuracy of nonlocal continuum modelling of SLGSs via MD simulations [240]. Reprinted with permission from Elsevier.

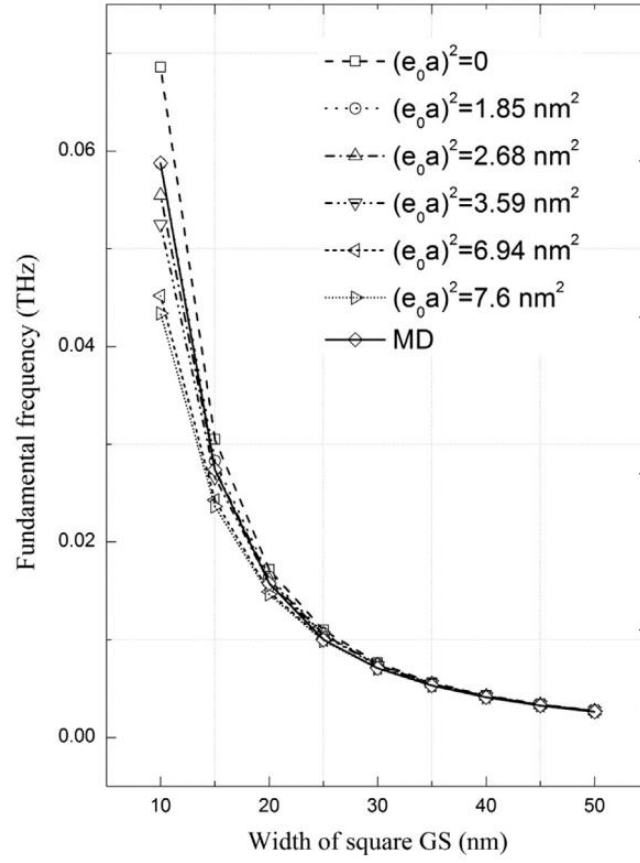


Fig. 26. Fundamental frequencies of SLGSs with four edges simply supported versus the width for different nonlocal parameters [271]. Reprinted with permission from Elsevier.

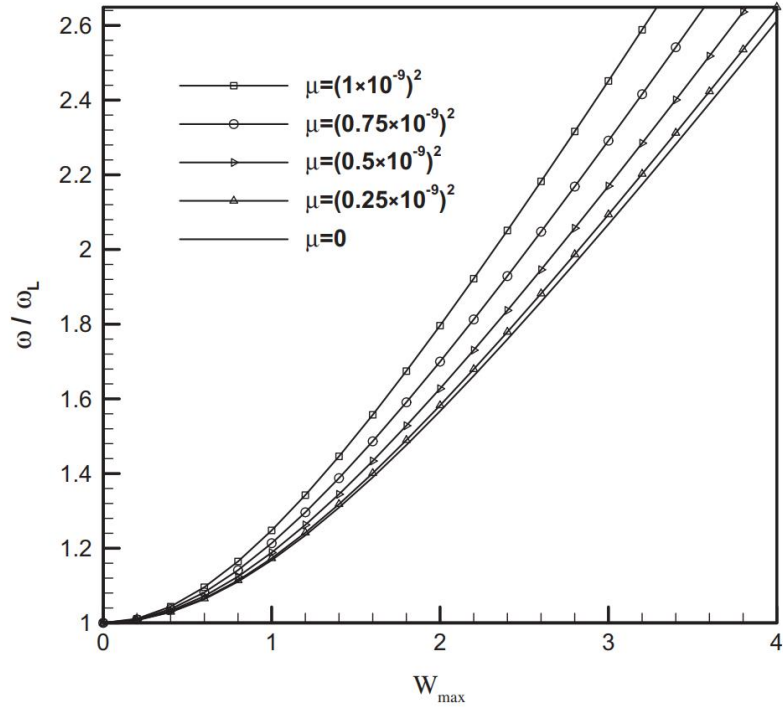


Fig. 27. Nonlinear frequency ratio of SLGSs against maximum deflection for different scale parameters

$(\mu = (e_0 a)^2)$  [280]. Reprinted with permission from Elsevier.

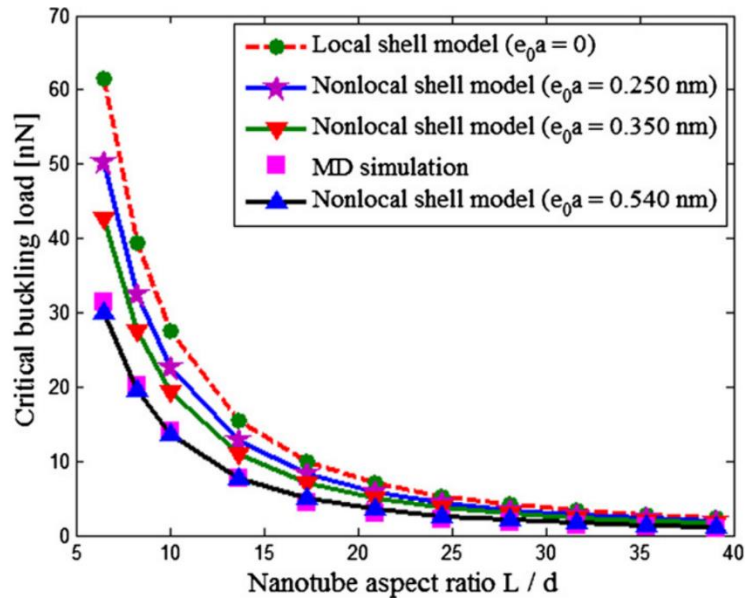


Fig. 28. Comparison between the nonlocal and local shell models for simply-supported SWCNTs [319]. Reprinted with permission from Elsevier.

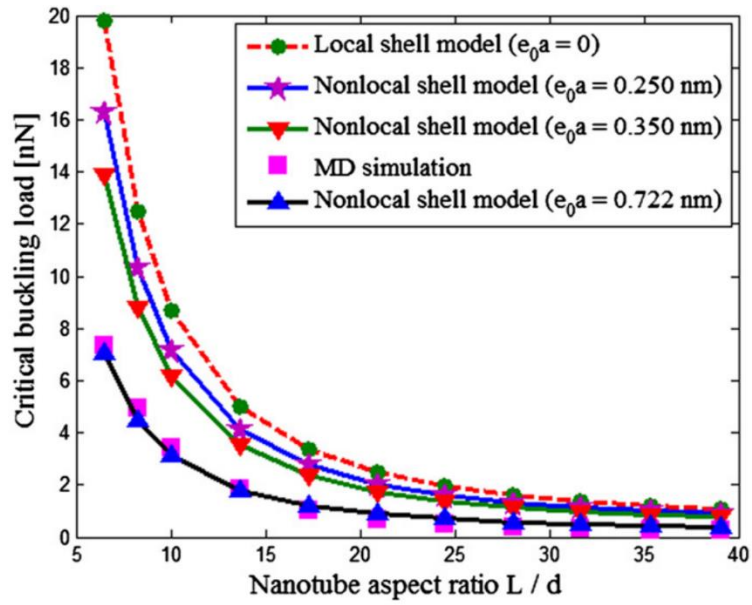


Fig. 29. Comparison between the nonlocal and local shell models for clamped-free SWCNTs [319]. Reprinted with permission from Elsevier.